ANSWERS: DIVISION M

SET #13

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 1,000,000	**2A.** 50	**3A.** 4	**4A.** 7	**5A.** 8
1B. 21	**2B.** 10	**3B.** 4	**4B.** 85	**5B.** $\frac{1}{45}$
1C. 15	**2C.** 108	**3C.** $\frac{12}{125}$	**4C.** 36	**5C.** 14
1D. 11	**2D.** 325	**3D.** 72	**4D.** 2	**5D.** 38
1E. 74	**2E.** $840	**3E.** a=6, b=2, c=3	**4E.** 25 sq units	**5E.** 300

SET #14

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 92	**2A.** .826	**3A.** 41	**4A.** 7	**5A.** 36
1B. 4	**2B.** 5	**3B.** 12	**4B.** $\frac{1}{3}$	**5B.** 23
1C. 39 sq mm	**2C.** 47	**3C.** $\frac{3}{5}$	**4C.** 9	**5C.** 56
1D. 5	**2D.** 3	**3D.** 82 units	**4D.** 27π sq cm	**5D.** 363
1E. $\frac{3}{8}$	**2E.** 25	**3E.** 108	**4E.** 48	**5E.** 7

SET #15

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 16	**2A.** 37, 73	**3A.** 11	**4A.** 4	**5A.** 9919
1B. 12	**2B.** 8	**3B.** 7	**4B.** $\frac{5}{7}$	**5B.** 47
1C. 720 sq mm	**2C.** 1000	**3C.** 19	**4C.** 12	**5C.** 7
1D. $125	**2D.** 165	**3D.** (3,10)	**4D.** 10 cm	**5D.** 54
1E. 5	**2E.** 11	**3E.** 43	**4E.** $\frac{3}{36}$	**5E.** 10π

SET #16

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 12	**2A.** Wednesday	**3A.** X=9, Y=3	**4A.** 21	**5A.** 100 miles
1B. 15	**2B.** 32 sq cm	**3B.** 10	**4B.** 10	**5B.** 2
1C. 1 & –11	**2C.** 61	**3C.** $\frac{2}{7}$	**4C.** 19,354	**5C.** –15
1D. 49	**2D.** 5	**3D.** 153	**4D.** 77	**5D.** 66
1E. 114	**2E.** $\frac{24}{5}$	**3E.** 4.8	**4E.** 126	**5E.** 19

SET 13 SOLUTIONS

Set 13 Olympiad 1

1A *Strategy: Rearrange the factors in a more convenient order.*

$125 \times 25 \times 5 \times 2 \times 4 \times 8$
$= (5\times2) \times (25\times4) \times (125\times8)$
$= 10 \times 100 \times 1000$
$=$ **1,000,000.**

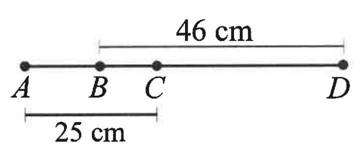
1B *Strategy: Look for perfect squares near the given number.*
Since $11^2 = 121$ and $12^2 = 144$, the least whole number value for N is the value for which $123 + N = 144$. **The least whole number N is 21.**

1C **METHOD 1:** *Strategy: Use the definition of even number.*
Because each of the two even numbers has a factor of 2, their product is a multiple of 4. Conversely, every multiple of 4 can be written as the product of two even numbers. There are 19 positive multiples of 4 that are less than 80, but we must exclude the 4 multiples that are 18 or less. **Thus, 15 numbers between 19 and 79 are the product of two even numbers.**

METHOD 2: *Factor every even number between 19 and 79.*
List the even numbers in the interval: 20, 22, 24, …, 74, 76, 78. Try factoring each into the product of 2 even numbers. Eliminate those that cannot be so factored. The remaining numbers are 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, and 76. In all, there are 15 possible products between 19 and 79.

> **Follow-Up:** *How many multiples of 3 are less than 500 and are not the product of two multiples of 3?* [111]

1D **METHOD 1:** *Strategy: First find the length* AB.
As shown, $AB + BC = 25$ cm and $BC + CD = 46$ cm. With BC common to both lengths, CD is 21 cm longer than AB. But $CD = 2.5 \times AB$. Then $(2.5 \times AB) - AB = 21$. Therefore, $1.5 \times AB = 21$ and the length $AB = 21 \div 1.5 = 14$. Finally, **the length of $\overline{BC}$ is** $25 - 14 =$ **11 cm.**



SET 13 SOLUTIONS

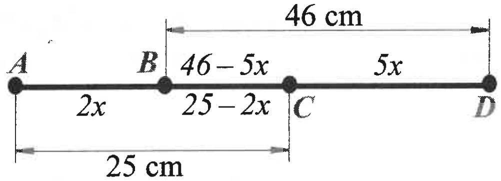
METHOD 2: *<u>Strategy</u>: Use possible lengths AB and CD to find BC.*

The table shows possible lengths CD and AB. Each pair is in the ratio of 5:2 and calculates the length BC in two ways. Only when CD is 35 and AB is 14 do both computations give the same value of BC. The length of $\overline{BC}$ is 11 cm.

CD	20	25	30	**35**	40
AB	8	10	12	**14**	16
BC* = 46 – *CD	26	21	16	11	6
BC* = 25 – *CD	17	15	13	11	9

METHOD 3: *<u>Strategy</u>: Use algebra.*

Because $CD{:}AB = 5{:}2$, represent the length CD by $5x$ and the length AB by $2x$. Then the length BC can be represented two ways, by $25 - 2x$ and by $46 - 5x$, as shown below.



From the diagram represent BC two ways:	$25 - 2x = 46 - 5x$
Add $5x$ to each side of the equation:	$25 + 3x = 46$
Subtract 25 from each side of the equation:	$3x = 21$
Divide each side of the equation by 3:	$x = 7$

Then $BC = 46 - 5(7) = 11$. Checking, $BC = 25 - 2(7) = 11$.

FOLLOW-UP: *Points A, B, C, D, and E lie on a straight line in the given order. The ratios of lengths are as follows — BC:AB = 4:3 and BC:CD = 2:1. If AE is 48 units and DE = BA, what is BC?* [16 units]

1E METHOD 1: *<u>Strategy</u>: Use counting principles.*

Emma buys a German novel and a Spanish novel, or a German novel and a French novel, or a Spanish novel and a French novel. She has 4 choices for the German novel and 5 for the Spanish novel, so she can buy novels in those two languages in $4 \times 5 = 20$ ways. Likewise, she can buy a German novel and a French novel in $4 \times 6 = 24$ ways, and she can buy a Spanish novel and a French novel in $5 \times 6 = 30$ ways. In all **Emma can purchase two novels in two languages in** $20 + 24 + 30 =$ **74 ways.**

METHOD 2: *Strategy: Make an organized list.*
Denote the 4 German novels as G_1, G_2, G_3, G_4, and similarly for the others.

If G_1 is one of the books, the other book can be S_1, S_2, S_3, S_4, S_5, *or* F_1, F_2, F_3, F_4, F_5, F_6 — that is 11 ways. Likewise, there are 11 ways if G_2 is chosen, 11 with G_3, and 11 with G_4 for a total of 44 ways.

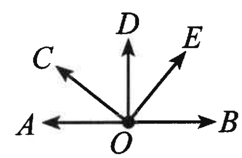
If no German novel is chosen, one book must be in Spanish and one in French.

If S_1 is one book, F_1, F_2, …, F_6 is the other: 6 ways. Continuing, there are 6 with S_2, etc., for a total of 30 ways. In all Emma can purchase the two novels in 44 + 30 = 74 ways.

> **FOLLOW-UP:** *(1) In how many ways can Emma purchase, in any order, two novels in the same language?* [31] *(2) In how many ways can she purchase, in any order, two novels in one language and one in each of the other two languages?*[720]

Olympiad 2

2A METHOD 1: *Strategy: First find m∠ COD.*



(1) $m\angle COB = 130$
$-\ m\angle DOB = 90$
$m\angle COD = 40$ →

(2) $m\angle COE = 90$
$-\ m\angle COD = 40$
$\mathbf{m\angle DOE = 50}$

METHOD 2: *Strategy: First find m∠AOC.*

(1) $m\angle AOB = 180$
$-\ m\angle COB = 130$
$m\angle AOC = 50$ →

(2) $m\angle AOD = 90$
$-\ m\angle AOC = 50$
$m\angle COD = 40$ →

(3) $m\angle COE = 90$
$-\ m\angle COD = 40$
$m\angle DOE = 50$

2B *Strategy: Use the distributive property.*

$$\frac{2.3 \times 2.01 + 3.7 \times 2.01}{0.3 \times 4.02} = \frac{(2.3 + 3.7) \times 2.01}{0.3 \times 4.02} = \frac{6.0 \times 2.01}{0.3 \times 4.02} = \frac{6.0}{0.3} \times \frac{2.01}{4.02} = 20 \times \frac{1}{2} = \mathbf{10.}$$

Variation: Notice that 4.02 is twice 2.01. Rewrite the denominator as 0.6 × 2.01.
Then $\frac{6.0 \times 2.01}{0.3 \times 4.02} = \frac{6.0 \times \cancel{2.01}}{0.6 \times \cancel{2.01}} = \frac{6.0}{0.6} = \frac{60}{6} = 10.$

2C METHOD 1: *Strategy: Combine the two cases.*
Suppose 2 students are absent and Mr. Alvarez still gives each student 4 sheets. He will have the original 16 sheets left over and in addition the 4 sheets that he would have given to each of the absentees. This total of 24 sheets is enough to give each of the students who are present 1 additional sheet, with 3 left over. Then there are 24 – 3 = 21 students present. **Mr. Alvarez has** 5 × 21 + 3 = **108 sheets of paper.**

METHOD 2: *Strategy: Use algebra.*

The class has N students registered and $N-2$ students present when 2 students are absent.

Then $4N+16$ = four sheets per student, with 16 sheets left over.
And $5(N-2)+3$ = five sheets per student when 2 students are absent with 3 sheets left over.

The number of sheets is the same whether or not the 2 students are absent:

	$5(N-2)+3$	$=4N+16$
Multiply $N-2$ by 5:	$5N-10+3$	$=4N+16$
Add −10 and 3:	$5N-7$	$=4N+16$
Add 7 to each side of the equation:	$5N$	$=4N+23$
Subtract $4N$ from each side of the equation:	N	$=23$

Then Mr. Alvarez started with $4\times 23+16=108$ sheets of paper.

> ***Follow-Up:** Find the least whole number N such that N is 1 more than a multiple of 3, N – 3 is 2 more than a multiple of 5, and N – 6 is 3 more than a multiple of 7.* [100; Hint: Find a number near N that is divisible by 3, 5, and 7.]

2D **METHOD 1:** *Strategy: Compare terms.*

The terms in Series A increase by 2. The terms in Series B increase by 4. If Series A is multiplied by 2 (row 2), its terms will also increase by 4.

Series A: $1+3+5+7+\ldots+21+23+25=169$
2 × Series A: $2+6+10+14+\cdots+42+46+50=338$
Series B: $1+5+9+13+\cdots+41+45+49=\ ?$

Each of the 13 terms in row 2 is 1 greater than the corresponding term in row 3. Therefore **the sum $1+5+9+\cdots+41+45+49$** $=338-13$ **is 325.**

METHOD 2: *Strategy: Use "Gaussian Addition".*

In the series $1+5+\cdots+49$, we get from 1 to 49 by adding 4 twelve times, so the series has 13 terms. Pair these terms as follows, working from the outside inward.

$(1+49)+(5+45)+(9+41)$, and so on. The sum of each pair is 50 and there are 6 pairs. The unpaired number is 25, the middle number. The sum is then $6\times 50+25=325$.

METHOD 3: *Strategy: Look for a pattern in the partial sums.*

The table at the right examines the sums of the first few terms. In each case, the sum is the product of the number of terms and the middle term (or the average of the 2 middle terms). Since the series has 13 terms, the sum we are looking for is $1+5+9+\ldots+41+45+49 = 13\times 25=325$.

Series	Sum	Sum, Factored
1	1	1×1
$1+5$	6	2×3
$1+5+9$	15	3×5
$1+5+9+13$	28	4×7
⋮	⋮	⋮

Other approaches are possible.
How many can you find?

SET 13 SOLUTIONS

Follow-Ups: (1) In the series 1 + 5 + 9 + ..., find the formula for the value of the n^{th} term. [$4n - 3$] *(2) What is the formula for calculating the sum of this series?* [$n(2n - 1)$; see table] *(3) If $x^3 = 3 \times 6 \times 12 \times 24 \times 48 \times 96$, what is the value of x?* [288]

2E METHOD 1: Strategy: *Use a frequency definition of probability.*
Consider a large and convenient number of days, say 100. Rain is expected for 40 days and fair weather for 60 days. Jess would expect to earn a total of $(40 \times \$1500) + (60 \times \$400) = \$84{,}000$. During the 100 day period, **Jess expects to earn** \$84,000, which is **an average of \$840 daily.**

METHOD 2: Strategy: *Pretend the average weather actually happens one day.*
Consider an "average" day. Assume it rains 40% of that day. During that time Jess earns 40% of \$1500, which is \$600 that day. It is fair the other 60% of that day, so Jess earns 60% of \$400, which is another \$240. Thus, on that "average" day, Jess expects to earn \$840.

Olympiad 3

3A METHOD 1: Strategy: *Write the numbers in standard form.*
$500{,}000{,}000 \div 170{,}000 = 50{,}000 \div 17$, which is between 2000 and 3000. **There are 4 digits to the left of the decimal point.**

METHOD 2: Strategy: *Use scientific notation.*
$500{,}000{,}000 \div 170.000 = (5 \times 10^8) \div (1.7 \times 10^5) = 2.\square\square \times 10^3$. Multiplying a one-digit number by 1000 produces 4 digits to the left of the decimal point.

3B Strategy: *Count the number of multiples of 7.*
Only multiples of 7 contain factors of 7. There are four multiples of 7 less than 30 and each contains 7 as a factor exactly once. Thus, **7 appears as a factor of the product 4 times.**

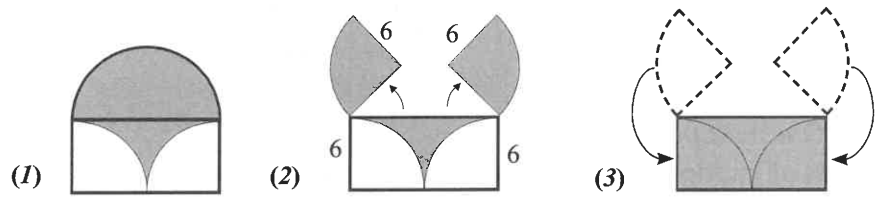
Follow-Ups: The product is often written as 30! ("30 factorial"). (1) How many factors of 3 does 30! have? [14] *(2) In part 1, why is the answer 14 instead of 10 or 13?* [9 and 18 each contain 3^2 and 27 contains 3^3.] *(3) 30! has how many factors of 10?* [7] *(4) In part 3, why is the answer 7 instead of 3?* [10 is not prime. Look for the number of times 5 appears as a factor.] *(5) In part 5, why do we use the factor 5 instead of the factor 2?*

SET 13 SOLUTIONS

3C *Strategy: Consider all three games.*
To win for the first time in the third game, Chloe must lose the first two games and then win the third. Since the probability that Chloe wins a game is $\frac{3}{5}$, then the probability that she does *not* win a game is $\frac{2}{5}$. By the multiplication principle, **the probability that Chloe loses the first two games and then wins the third is $\frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{125}$.**

FOLLOW-UPS: (1) Using the same information, what is the probability that Chloe wins for the second time in the third game? $\left[\frac{36}{125}\right]$ *(2) What is the probability that Chloe wins at least 2 games?* $\left[\frac{81}{125}\right]$

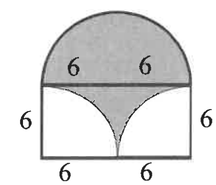
3D METHOD 1: *Strategy: Rearrange the regions more conveniently.*



The area of the shaded region is the same as the area of the rectangle, which is $6 \times 12 =$ **72 sq cm.**

METHOD 2: *Strategy: Add and subtract areas.*
The radius of the given semicircle is 6 cm.

From the sum of the areas of the given semicircle and the rectangle, subtract the areas of the two quarter circles:



The area of the given semicircle:

$$[\pi 6^2 \div 2] \text{ sq cm} + [12 \times 6] \text{ sq cm} - [(\pi 6^2 \div 4) \times 2] \text{ sq cm} =$$
$$18\pi \text{ sq cm} + 72 \text{ sq cm} - 18\pi \text{ sq cm} =$$
$$72 \text{ sq cm.}$$

The area of the shaded region is 72 sq cm.

3E *Strategy: Write the fraction as a mixed number.*

$$\frac{45}{7} = 6 + \frac{3}{7} = a + \cfrac{1}{b + \cfrac{1}{c}}, \text{ so } a = 6.$$

Now write $\frac{3}{7}$ so that its numerator is 1: $\frac{3}{7} = \cfrac{1}{\frac{7}{3}} = \cfrac{1}{b + \frac{1}{c}}$

Next examine b and c: $\frac{7}{3} = 2 + \frac{1}{3} = b + \frac{1}{c}$, so $b = 2$ and $c = 3$.

Thus, $a = 6$, $b = 2$, and $c = 3$.

SET 13 SOLUTIONS

Olympiad 4

4A **METHOD 1:** *Strategy: Express each side as the product of factors.*
Rewrite each side of the equation as the product of common factors.

$$52 \times 50 \times N = 40 \times 13 \times 35$$
$$4 \times 13 \times 5 \times 10 \times N = 4 \times 10 \times 13 \times 5 \times 7$$
$$4 \times 13 \times 5 \times 10 \times N = 4 \times 13 \times 5 \times 10 \times 7$$

Notice the common factors of 4, 13, 5 and 10 on each side. ***N* is 7.**

METHOD 2: *Strategy: Use algebra.*
Do the multiplication to get $2600N = 18{,}200$. Divide both sides of the equation by 2600. $N = 7$.

> **FOLLOW-UP:** *Find the least common multiple of 520 and 280.* [3640]

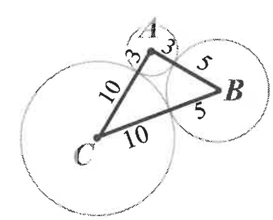
4B *Strategy: List the multiples of each.*
Write two lists of multiples between 0 and 100, as shown below.

Multiples of 17: 17, 34, 51, 68, **85**, …
Multiples of 21: 21, 42, 63, **84**, …

The only pair of consecutive integers is 84 and 85. Therefore, **the greater of the two consecutive integers is 85.**

> **FOLLOW-UPS:** *(1) Without continuing the lists, how could you find other pairs of consecutive integers such that one is a multiple of 17 and the other of 21?* [Add 17×21 to both 84 and 85 as often as desired.] *(2) Is it possible for two multiples of 15 and 21 to be consecutive integers? Explain.* [No. With a common factor of 3, every pair must differ by a multiple of 3.]

4C *Strategy: Find the radii of the circles.*
The area of a circle is given by the formula $A = \pi r^2$, so circles with areas of 9π, 25π, and 100π have radii of 3, 5, and 10 respectively. The lengths of the sides of the triangle are found by adding pairs of radii together. Therefore, AB = 8 cm, BC = 15 cm, and CA = 13 cm. **The perimeter of the triangle is** 8 + 15 + 13 = **36 cm.**



SET 13 SOLUTIONS

4D **METHOD 1:** *Strategy: Use the multiplication algorithm.*

Start with the multiplication on the left. Each partial product contains only 2 digits. First, q is 1, so C = 7 and pq represents 91. In the tens column, (9 + s) ends in 1. Then s is 2, so B = 4 and rs represents 52. This yields the multiplication on the right.

$$\begin{array}{rrrr} & & 1 & 3 \\ \times & A & B & C \\ \hline & & p & q \\ & r & s & \\ t & u & & \\ \hline 3 & \square & 1 & 1 \end{array} \qquad \begin{array}{rrrr} & & 1 & 3 \\ \times & A & 4 & 7 \\ \hline & & 9 & 1 \\ & 5 & 2 & \\ t & u & & \\ \hline 3 & \square & 1 & 1 \end{array}$$

Since the thousands digit of the product is 3, t is 2 or 3 and A is 2 or 3. If A is 3, tu represents 39 and the final product is 4511, not 3□11. Thus A is 2, tu represents 26 and the final product is 3211. **The missing digit is 2.**

METHOD 2: *Strategy: Start with an arbitrary value.*

Divide 3011 by 13 to get a quotient of 231 and a remainder of 8. Since 100 divided by 13 leaves a remainder of 9, each time we increase 3011 by 100, we add 9 to that remainder value of 8: 8, 17, 26, … and look for a multiple of 13. Since 26 is a multiple of 13 and is obtained by adding 2 nines to 8, then 3011 + 200 = 3211 will be a multiple of 13. The number is 3211 and the missing digit is 2.

Dividend	Remainder
3011	**8**
3111	8 + 9 = **17**
3211	8 + 18 = **26**

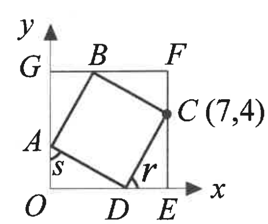
METHOD 3: *Strategy: Use a divisibility test.*

There are several tests for divisibility by 13. Here are two:

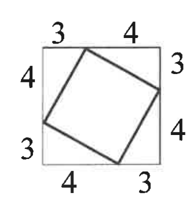
1. Start at the right and split the digits into groups of 3. Alternately add and subtract the groups until a 3 digit number remains. The original number is divisible by 13 if and only if the final 3 digit number is. Here, 3□11 is divisible by 13 only if □11 – 3 = □08 is. Use Method 2 to find that the missing digit is 2.
2. Multiply the units digit by 4 and add it to the original number without the units digit. Continue in this fashion until a 2 digit number remains. The original number is divisible by 13 if and only if the final 2 digit number is. Here 3□11 → 3□1 +4(1) = 3□5. 3□ + 4(5) = 5□. Only if the missing digit is 2 is 5□ a multiple of 13.

4E *Strategy: Wrap the square in a box.*

Draw ΔDEC congruent to right ΔAOD as shown. $\angle r \cong \angle s$ and $\angle O \cong \angle ADC$, so the sum of the three angles at D equals the sum of the three angles in ΔAOD (180°). Thus $\overline{ODE}$ is a straight line segment and E is on the x-axis. Construct 2 more right triangles congruent to ΔAOD, as shown, to produce square $OEFG$.



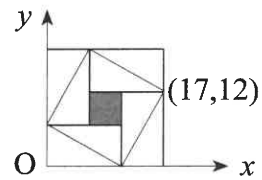
Because C is at (7,4), $OE = 7$ and $CE = 4$. Then $OD = 4$ and $DE = 3$. Since the four triangles are congruent, the legs in each triangle have lengths of 3 and 4.



SET 13 SOLUTIONS

METHOD 1: *Strategy: Use known areas.*
OEFG is a square of side 7, so its area is 49. The area of each of the four right triangles is ½(3)(4) = 6. **The area of the square *ABCD* is** then 49 – 4 × 6 = **25.**

METHOD 2: *Strategy: Use the Pythagorean Theorem.*
In right ΔCDE, $CD^2 = 3^2 + 4^2$. So $CD = 5$, and the area of the square $ADCB = 5^2 = 25$.



Follow-Up: (1) Find the area of the shaded square in the center of the largest square in this picture. [49] (2) *Find the area of a triangle whose vertices are A(0,0), B (4,5) and C(2,6).* [7 sq units; wrap the triangle in a rectangle and subtract areas.]

Olympiad 5

5A *Strategy: Use the definition of the operation.*
2 ✵ 6 = 2 + 3 × 6 = 20 and *N* ✵ 4 = *N* + 3 × 4 = *N* + 12
Then 20 = *N* + 12 and **the value of *N* is 8.**

5B **METHOD 1:** *Strategy: Extend the process of "cancellation".*
"Cancel" identical numerators and denominators with each other (that is, divide out each identical common factor greater than 1). This can be done 6 times. Then we are left with $\frac{2}{90}$.
In simplest terms, the product is $\frac{1}{45}$.

METHOD 2: *Strategy: Multiply all fractions and then simplify.*
The product of all the numerators is 40,320.
The product of all the denominators is 1,814,400.
1,814,400 ÷ 40,320 = 45. In simplest terms, the product is $\frac{1}{45}$.

5C *Strategy: Simplify the sum.*
Start with a sum of zero: The sum $^{-}10 + {}^{-}9 + {}^{-}8 + \cdots + 8 + 9 + 10 = 0$. Continue to add integers starting with 11 until the desired sum is obtained. Because 11 + 12 + 13 + 14 = 50, ***N* = 14.**

SET 13 SOLUTIONS

5D METHOD 1a: <u>Strategy</u>: *Consider the number of wins.*
The following table shows that the Pumas won 2 of their first 9 games and then 3 of every 4 games on average. They ended with 2 wins in every 3 games.

Games won	2	5	8	...	35	38
Games played	9	13	17	...	53	57

Since only $\frac{38}{57}$ simplifies to $\frac{2}{3}$, **the Pumas won 38 games in all.**

METHOD 1b: <u>Strategy</u>: *Consider the number of losses.*
The following table shows that the Pumas lost 7 of their first 9 games and then lost 1 of every 4 games on average. They ended with 19 losses in 57 games, which is 1 loss in every 3 games.

Games lost	7	8	9	...	18	19
Games played	9	13	17	...	53	57

Since they lost a total of 19 games, the Pumas won 38 games in all.

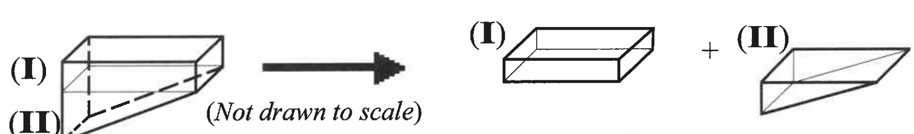
METHOD 2: <u>Strategy</u>: *Use algebra.*
Let w = ratio factor. Then they won $3w$ and lost $4w$ of the remaining games.
In all the Pumas won $2 + 3w$ games and played $9 + 4w$ games.

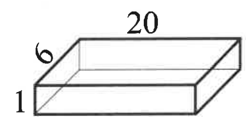
$$\frac{2+3w}{9+4w} = \frac{2}{3}$$

Cross-multiply: $3(2+3w) = 2(9+4w)$
Multiply out on each side of the equation: $6 + 9w = 18 + 8w$
Subtract $8w$ from each side of the equation: $6 + w = 18$
Subtract 6 from each side of the equation: $w = 12$
Now find the value of $2 + 3w$: $2 + 3w = 38$

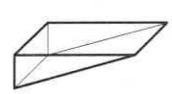
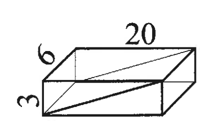
The Pumas won 38 games in all.

5E METHOD 1: <u>Strategy</u>: *Split the figure into more familiar shapes and add.*





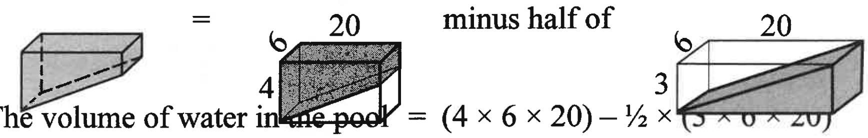
(I) Volume of **I** = 6 × 20 × 1 = 120 cu m:

(II) Volume of **II** = ½ × 6 × 20 × 3 = 180 cu m:  is half of 

The pool can hold 120 + 180 = **300 cubic meters of water.**

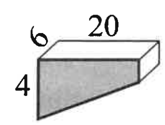
SET 13 SOLUTIONS

METHOD 2: <u>Strategy</u>: *Embed the figure in a more familiar shape and subtract.*
Box in the pool and compute the volume of the resulting rectangular solid. Next, find and subtract the volume of the extra "wedge" you added.



The volume of water in the pool $= (4 \times 6 \times 20) - \frac{1}{2} \times (3 \times 6 \times 20)$
$= 480 - 180$
$= 300$ cu m

METHOD 3: <u>Strategy</u>: *Use formulas.*
In the diagram, consider the shaded side of the pool as the base of a prism. The volume of the prism is $V = Bh$, where B is the area of the base and h is the height of the prism.

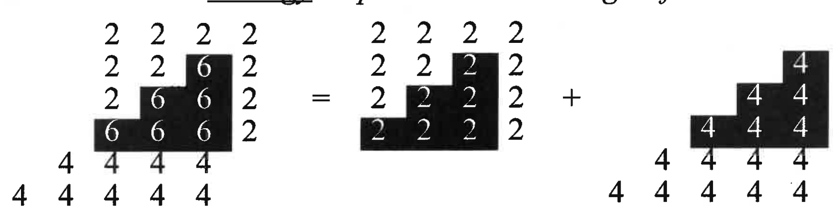


The base is a trapezoid, and its area is given by $\frac{1}{2}h(b_1 + b_2)$, where h is the height of the trapezoid and b_1 and b_2 are the lengths of its bases. The area of this trapezoid is $B = \frac{1}{2}(20)(4+1) = 50$ sq m, and the volume of the pool is $Bh = 50 \times 6$. The pool can hold 300 cu m of water.

Set 14 Olympiad 1

1A **METHOD 1:** <u>Strategy</u>: *Count the number of times each digit appears.*
Multiply each value by the number of times it appears. Add those products.
$(10 \times 2) + (9 \times 4) + (6 \times 6) = 92$. **The sum is 92.**

METHOD 2: <u>Strategy</u>: *Separate into a rectangle of 2s and a triangle of 4s.*



The sum is $(16 \times 2) + (15 \times 4) = 92$.

1B **METHOD 1:** <u>Strategy</u>: *Compare each coin to the average value.*
Each nickel is worth 5 cents less than the average; each quarter is worth 15 cents more. Combine each quarter with 3 nickels for an average value of 10 cents. There are 12 nickels, so **4 quarters must be added to the collection.**