## Mathcounts

Things to Know

## 1. Primes less than 200

$2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97,101,103$, $107,109,113,127,131,137,139,149,151,157,163,167,173,179,181,191,193,197,199$
2. Perfect squares less than 1,000

$$
\begin{array}{lllllll}
0^{2}=0 & 5^{2}=25 & 10^{2}=100 & 15^{2}=225 & 20^{2}=400 & 25^{2}=625 & 30^{2}=900 \\
1^{2}=1 & 6^{2}=36 & 11^{2}=121 & 16^{2}=256 & 21^{2}=441 & 26^{2}=676 & 31^{2}=961 \\
2^{2}=4 & 7^{2}=49 & 12^{2}=144 & 17^{2}=289 & 22^{2}=484 & 27^{2}=729 & \\
3^{2}=9 & 8^{2}=64 & 13^{2}=169 & 18^{2}=324 & 23^{2}=529 & 28^{2}=784 & \\
4^{2}=16 & 9^{2}=81 & 14^{2}=196 & 19^{2}=361 & 24^{2}=576 & 29^{2}=841 &
\end{array}
$$

3. Powers of natural numbers

$$
\begin{array}{llllllll}
2^{0}=1 & 3^{0}=1 & 4^{0}=1 & 5^{0}=1 & 6^{0}=1 & 7^{0}=1 & 8^{0}=1 & 9^{0}=1 \\
2^{1}=2 & 3^{1}=3 & 4^{1}=4 & 5^{1}=5 & 6^{1}=6 & 7^{1}=7 & 8^{1}=8 & 9^{1}=9 \\
2^{2}=4 & 3^{2}=9 & 4^{2}=16 & 5^{2}=25 & 6^{2}=36 & 7^{2}=49 & 8^{2}=64 & 9^{2}=81 \\
2^{3}=8 & 3^{3}=27 & 4^{3}=64 & 5^{3}=125 & 6^{3}=216 & 7^{3}=343 & 8^{3}=512 & 9^{3}=729 \\
2^{4}=16 & 3^{4}=81 & 4^{4}=256 & 5^{4}=625 & 6^{4}=1296 & 7^{4}=2401 & & \\
2^{5}=32 & 3^{5}=243 & 4^{5}=1024 & & & & & \\
2^{6}=64 & 3^{6}=729 & & & & & \\
2^{7}=128 & & & & & & \\
2^{8}=256 & & & & & \\
2^{9}=512 & & & & & \\
2^{10}=1024 & & & & & \\
\end{array}
$$

4. Fractions and Decimal Equivalents

$$
\begin{array}{lll}
\frac{1}{2}=.5 & \frac{1}{6}=.1 \overline{6} & \frac{1}{11}=. \overline{09} \\
\frac{1}{3}=. \overline{3} & \frac{5}{6}=.8 \overline{3} & \frac{2}{11}=. \overline{18} \\
\frac{2}{3}=. \overline{6} & \frac{1}{7}=. \overline{142857} & \\
\frac{1}{4}=.25 & \frac{1}{8}=.125 & \\
\frac{3}{4}=.75 & \frac{3}{8}=.375 & \\
\frac{1}{5}=.2 & \frac{5}{8}=.625 & \\
\frac{2}{5}=.4 & \frac{7}{8}=.875 & \\
\frac{3}{5}=.6 & \frac{1}{9}=. \overline{1} & \\
\frac{4}{5}=.8 & \frac{2}{9}=. \overline{2} &
\end{array}
$$

5. Factorials

$$
\begin{aligned}
& 0!=1 \\
& 1!=1 \\
& 2!=2 \\
& 3!=6 \\
& 4!=24 \\
& 5!=120 \\
& 6!=720 \\
& 7!=5040 \\
& 8!=40,320 \\
& 9!=362,880 \\
& 10!=3,628,800
\end{aligned}
$$

## 6. Combinatorics Formulas

${ }_{n} C_{r}=\frac{n!}{r!(n-r)!}$ (when order does not matter)
${ }_{n} P_{r}=\frac{n!}{(n-r)!}$ (when order does matter)

## 7. Divisibility rules

$2 \rightarrow$ iff units digit is even
$3 \rightarrow$ iff sum of digits is divisible by 3
$4 \rightarrow$ iff last two digits form a number divisible by 4
$5 \rightarrow$ iff units digit is 0 or 5
$6 \rightarrow$ iff number is divisible by both 2 and 3
$7 \rightarrow$ iff result of subtracting twice the last digit from the number remaining when the last digit is removed is divisible by 7
$8 \rightarrow$ iff last three digits of number form a number divisible by 8
$9 \rightarrow$ iff sum of digits is divisible by 9
$10 \rightarrow$ iff units digit is 0
$11 \rightarrow$ iff result of alternately adding and subtracting the digits is divisible by 11 .
$12 \rightarrow$ iff number is divisible by both 4 and 3.
8. Special Factorizations
$a^{2}+2 a b+b^{2}=(a+b)^{2}$
$a^{2}-2 a b+b^{2}=(a-b)^{2}$
$a^{3}+3 a^{2} b+3 a b^{2}+b^{3}=(a+b)^{3}$
$a^{3}-3 a^{2} b+3 a b^{2}-b^{3}=(a-b)^{3}$
$a^{2}-b^{2}=(a+b)(a-b)$
$a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)$
$a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$

## 9. Quadratic Formula

If $a x^{2}+b x+c=0$, then

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

10. Units Digits Patterns $\left(a^{1}, a^{2}, a^{3} ..\right)$

| $1 \rightarrow 1$ | $6 \rightarrow 6$ |
| :--- | :--- |
| $2 \rightarrow 2,4,8,6$ | $7 \rightarrow 7,9,3,1$ |
| $3 \rightarrow 3,9,7,1$ | $8 \rightarrow 8,4,2,6$ |
| $4 \rightarrow 4,6$ | $9 \rightarrow 9,1$ |
| $5 \rightarrow 5$ |  |

11. Geometry

## General

Things that may be assumed from a geometric figure:

1. Lines that appear to be straight are straight.
2. Points of intersection are depicted accurately.
3. Points shown on a line are collinear.
4. All the points in the figure are coplanar unless specified otherwise.
5. Relative positions of points are accurate.

Some that may not be assumed from a given geometric figure:

1. Exact measurement or relative size of figures.
2. Congruence or similarity.
3. Parallel or perpendicular lines.

## Miscellaneous

- Number of diagonals in an $n$-gon $\rightarrow \frac{n(n-3)}{2}$
- Sum of measures of internal angles in an $n$-gon $\rightarrow 180^{\circ}(n-2)$
- Sum of measures of external angles in an $n$-gon $\rightarrow 360^{\circ}$
- If the ratio of the same linear dimension of two similar two-dimensional figures is $k$, then the ratio of their areas is $k^{2}$
- If the ratio of the same linear dimension of two similar three-dimensional figures is $k$, then the ratio of their volumes is $k^{3}$


## Circle

- Area $=\pi r^{2}$, Circumference $=2 \pi r$, where $r$ is the radius


## Triangle

- Area $=\frac{1}{2} b h$, where $b$ is a side (the base), and $h$ is the length of the altitude drawn to that side (the height)
- Area $=\sqrt{s(s-a)(s-b)(s-c)}$, where $a, b, c$ are the side lengths and $s$ is the semiperimeter (Heron's Formula)
- Area $=\frac{1}{2} a b \sin C$, where $a, b$ are two sides and $C$ is the angle formed by those sides
- Area $=r s$, where $r$ is the inradius of the triangle and $s$ is the semiperimeter
- Area $=\frac{s^{2} \sqrt{3}}{4}$, where $s$ is the length of a side of an equilateral triangle


## Quadrilaterals

- Parallelogram
- Area $=b h$, where $b$ is the base and $h$ is the height of the parallelogram
- Rectangle
- Area $=l w$, Perimeter $=2(l+w)$, where $l$ and $w$ are the length and width of the rectangle
- Square
- Area $=s^{2}$, where $s$ is the side length
- Area $=d^{2} / 2$, where $d$ is the diagonal
- Rhombus
- Area $=\frac{1}{2} d_{1} d_{2}$, where $d_{1}$ and $d_{2}$ are the two diagonals of the rhombus


## Prism

- Volume $=B h$, where $B$ is the area of the base and $h$ is the height
- Surface area $=2 B+P h$, where $B$ is the area of the base, $P$ is the perimeter of the base, and $h$ is the height of the prism

Pyramid

- Volume $=\frac{1}{3} B h$, where $B$ is the area of the base and $h$ is the height


## Rectangular Prism

- Volume=lwh
- Surface area $=2(l w+w h+l h)$
- Length of space diagonal $=\sqrt{l^{2}+w^{2}+h^{2}}$, where $l, w$, and $h$ are the lengths of the edges of the prism


## Cube

- Volume $=e^{3}$
- Surface Area $=6 e^{2}$
- Length of Space Diagonal: $e \sqrt{3}$, where $e$ is the edge length


## Cylinder

- Volume $=\pi r^{2} h$
- Surface Area $=2 \pi r h+2 \pi r^{2}$, where $r$ is the radius and $h$ is the height


## Cone

- Volume $=\frac{1}{3} \pi r^{2} h$
- Surface Area $=\pi r^{2}+\pi r \sqrt{r^{2}+h^{2}}$


## Sphere

- Volume $=\frac{4}{3} \pi r^{3}$
- Surface Area $=4 \pi r^{2}$, where $r$ is the radius


## 12. Triangles

Triangle Inequality
$a+b<c$, where $a$ and $b$ are the shorter sides and $c$ is the largest side
Pythagorean Theorem
$a^{2}+b^{2}=c^{2}$, where $a$ and $b$ are the legs and $c$ is the hypotenuse of a right triangle
Law of Cosines
$c^{2}=a^{2}+b^{2}-2 a b \cos C$, where $a, b$, and $c$ are the sides of a triangle, and $C$ is the measure of the angle formed by sides $a$ and $b$

## Congruency and Similarity Theorems

- Congruency

For any triangles: SSS, SAS, ASA, AAS
For right triangles: HL, LL, SA
Similarity
For any triangles: AA, SAS, SSS

## Pythagorean Triples

- $3,4,5$
- $5,12,13$
- $8,15,17$
- $7,24,25$
- $12,35,37$
- $9,40,41$
- $k a, k b, k c$, where $(a, b, c)$ is a Pythagorean triple and $k$ is any positive real number


## 13. Trigonometry

Basic trig function mnemonic device: "SOHCAHTOA"

Definitions of trigonometric functions
$\sin \theta=\frac{\text { opposite }}{\text { hypotenuse }}$
$\cos \theta=\frac{\text { adjacent }}{\text { hypotenuse }}$
$\tan \theta=\frac{\text { opposite }}{\text { adjacent }}=\frac{\sin \theta}{\cos \theta}$
$\sec \theta=\frac{\text { hypotenuse }}{\text { adjacent }}=\frac{1}{\cos \theta}$
$\csc \theta=\frac{\text { hypotenuse }}{\text { opposite }}=\frac{1}{\sin \theta}$
$\cot \theta=\frac{\text { adjacent }}{\text { opposite }}=\frac{1}{\tan \theta}$
Values of trigonometric functions for common angle measures

| $\theta$ | $0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 1 |
| $\tan \theta$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | $\infty$ |

## 14. Some Approximations

- $\pi \approx 3.14159$
- $\sqrt{2} \approx 1.414$
- $\sqrt{3} \approx 1.732$
- $\sqrt{5} \approx 2.236$


## 15. Arithmetic Sequences

- $a_{1}+a_{2}+\cdots+a_{n}=\frac{n\left(a_{1}+a_{n}\right)}{2}$
- $1+2+3+\cdots+n=\frac{n(n+1)}{2}$
- $1+3+5+\cdots+(2 n+1)=n^{2}$
- $2+4+6+8+\cdots+(2 n)=n(n+1)$


## 16. Conversion Factors

| Length and Area |
| :--- |
| $12 \mathrm{in} .=1 \mathrm{ft}$. |
| $3 \mathrm{ft} .=1 \mathrm{yd}$. |
| $5280 \mathrm{ft} .=1 \mathrm{mi}$. |
| $1760 \mathrm{yd} .=1 \mathrm{mi}$. |
| $2.54 \mathrm{~cm}=1 \mathrm{in}$. |
| 640 acres $=1 \mathrm{mi.}^{2}$ |
| Weight |
| 16 oz. |$=1 \mathrm{lb} . \quad$.

Volume
$3 \mathrm{ts} .=1 \mathrm{tbsp}$.
2 tbsp. $=1$ fl. oz.
$8 \mathrm{fl} . \mathrm{oz}=1$ cup
2 cups $=1$ pt.
$2 \mathrm{pt} .=1 \mathrm{qt}$.
$4 \mathrm{qt} .=1$ gal.
$1 \mathrm{~mL}=1 \mathrm{~cm}^{3}$

| Metric Prefixes |  |
| :--- | :--- |
| kilo- | $10^{3}$ |
| hecto- | $10^{2}$ |
| deca- | $10^{1}$ |
| -- | $10^{0}$ |
| deci- | $10^{-1}$ |
| centi- | $10^{-2}$ |
| milli- | $10^{-3}$ |

