

22. We're told that  $10d+5n = 25(n-d)$ , so  $10d+5n = 25n-25d$ , from which  $35d = 20n$ . Dividing by  $35n$ ,  $d:n = 20:35 = 4:7$ .

- A) 4:7 B) 7:4 C) 1:2 D) 2:1



23. From the 99 integers  $\{1, \dots, 99\}$ , choose  $n$  different integers. Since  $100 = 1 \times 2 \times 5 \times 10$ , the greatest possible value of  $n$  is 4.

- A) 2 B) 3 C) 4 D) 5

24. Undefined  $\Leftrightarrow x+2006 = 0$  or  $x + \frac{2005}{x+2006} = 0$ . In the latter case,  $x^2+2006x+2005 = (x+2005)(x+1) = 0$ ; so  $x = -2006, -2005, -1$ .

- A) 1 B) 2 C) 3 D) 4

25. The smallest such number is the square root of  $4^{144}$ , which is  $4^{72}$ .

- A)  $4^{12}$  B)  $4^{72}$  C)  $4^{144}$  D)  $4^{288}$

26.  $x-y + \frac{1}{x+y} = \frac{(x-y)(x+y)+1}{x+y} = \frac{x^2-y^2+1}{x+y}$ , so  $\diamond = -y^2$ .

- A)  $-y^2$  B)  $xy$  C)  $y^2$  D)  $-xy$

27. The roots of  $x^2 = a^2$  are roots of  $(x^2)^2 = (a^2)^2 \Leftrightarrow x^4 - a^4 = 0$ .

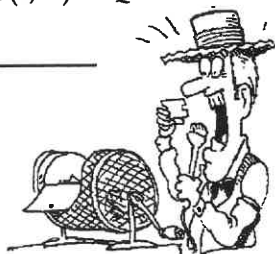
- A)  $x-a = 0$  B)  $x+a = 0$  C)  $x^2+a^2 = 0$  D)  $x^4-a^4 = 0$

28. Try  $(-2,1)$  in QII. Change both signs to get  $(2,-1)$  in QIV.

- A) I B) II C) III D) IV

29. The whole-number factors of 64 are  $1, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6$ . The product of these is  $1 \times 2^1 \times 2^2 \times 2^3 \times 2^4 \times 2^5 \times 2^6 = 2^{1+2+3+4+5+6} = 2^{21}$ .

- A)  $2^{36}$  B)  $2^{21}$  C)  $2^{12}$  D)  $2^{11}$



30. Given  $\Leftrightarrow 1 < |x+1| \Leftrightarrow x+1 < -1$  or  $x+1 > 1 \Leftrightarrow x < -2$  or  $x > 0$ .

- A)  $x < -2$  B)  $x > -1$  C)  $x < 0$  D)  $x < 1$



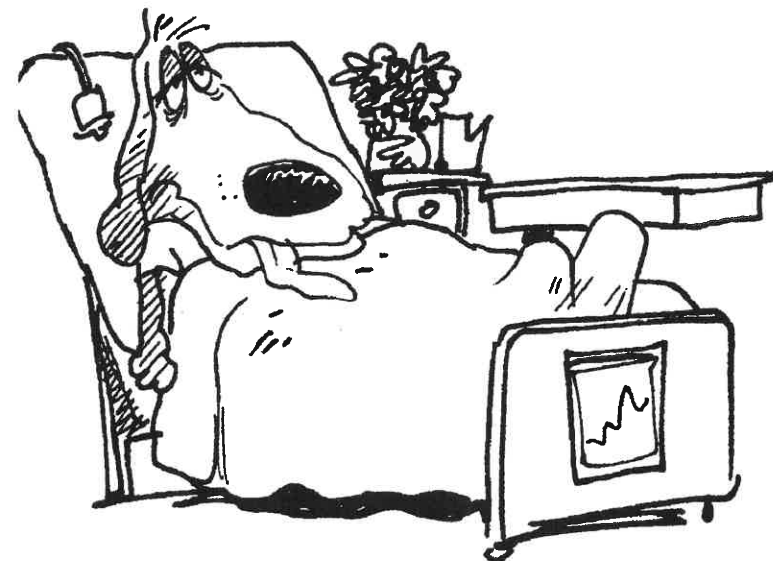
Information & Solutions

Spring, 2006

Contest Information

A

- **Solutions** Turn the page for detailed contest solutions (written in the question boxes) and letter answers (written in the *Answers* column to the right of each question).
- **Scores** Please remember that *this is a contest, not a test*—and there is no “passing” or “failing” score. Few students score as high as 24 points (80% correct). Students with half that, 12 points, *deserve commendation!*
- **Answers & Rating Scale** Turn to page 152 for the letter answers to each question and the rating scale for this contest.



The end of the contest A

1.	A	$1. (2+0+0+6)^{(2 \times 0 \times 0 \times 6)} = 8^0 = 1.$
2.	D	2. Given $= (-2)(1)(-2)(2)(-2)(3)(-2)(4)(-2)(5) = (1)(2)(3)(4)(5)(-32).$
3.	D	3. Since $(x+1)+(2x+2) = 3x+3 = 3(x+1),$ $(x+1)+(2x+2)$ scoops cost $3 \times \$3 = \$9.$ A) \$18 B) \$15 C) \$12 D) \$9
4.	B	4. $8 = \frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{x+2} = x-2.$ A) 10 B) 8 C) 6 D) 4
5.	C	5. $(10x)_{100} = 10^{100}x_{100} = (10x^{100})_{(10^9)}.$ A) 1 B) 10 C) $10^{99}$ D) $10^{100}$
6.	C	6. If $x = 0.5$ , then $x^2 - x = (0.5)^2 - 0.5 = 0.25 - 0.5 = -0.25 < 0.$ A) 2 B) 1 C) 0.5 D) -0.5
7.	B	7. Clearing fractions, $\frac{3}{x} < 3 \Leftrightarrow x < 9.$ Since $x > 0, x = 1, 2, \dots, 8.$ A) 9 B) 8 C) 6 D) 3
8.	B	8. Since area $= lw, l:w = 48:1$ or $24:2$ , or $16:3$ , or $12:4$ , or $8:6.$ Choice B is not included among these possibilities. A) 3:1 B) 6:1 C) 12:1 D) 48:1
9.	C	9. The ratio of the number of teams that wear red jerseys to the number that wear blue is 7:13. Of 20 teams, 7 wear red jerseys, so the percent that wear red jerseys is $7/20 = 35/100 = 35\%.$ A) 70% B) 49% C) 35% D) 20%
10.	B	10. The first multiple of $x^2$ that's divisible by $2x$ is $2x^4.$ A) $2x$ B) $2x^2$ C) $2x^3$ D) $2x^4$
11.	D	11. $x^{18} + 2x^{17} + x^{16} = x^{16}(x^2 + 2x + 1) = x^{16}(x+1)^2.$ A) $x^4$ B) $x^8$ C) $x^{12}$ D) $x^{16}$



12.	D	12. I was in the hospital for $ 2x  +  -x $ days $= 2 x  +  x $ days. $= 3 x $ days. A) $3x$ B) $x$ C) $ x $ D) $3 x $
13.	A	13. Rewrite each choice as shown below. Choice A is largest if $x > 1.$ A) $x_{50}$ B) $x_{20}$ C) $x_{10}x_5 = x_{15}$ D) $x^2x_{25} = x_{27}$
14.	D	14. $x = \pm\sqrt{2006}$ . The product is $-\sqrt{2006} \times \sqrt{2006} = -2006.$ A) 2006 B) 5002 C) $2\sqrt{2006}$ D) -2006
15.	A	15. Each side has length $(x+1)^2$ . Perimeter $= 4s = 4(x+1)^2.$ A) $4(x+1)^2$ B) $(x+1)^2$ C) $4(x+1)$ D) $4(x+1)$
16.	B	16. $\frac{w^2w^4w^6 \times \dots \times w^{48}w^{50}}{w^1w^3w^5 \times \dots \times w^{47}w^{49}} = \frac{w^1}{w^2} \times \frac{w^3}{w^4} \times \dots \times \frac{w^{49}}{w^{50}} = w^{-\frac{1}{50}}.$ A) $w^{24}$ B) $w^{25}$ C) $w^{49}$ D) $w^{50}$
17.	B	17. Any positive integer that's both a perfect square and a perfect cube is of the form $n^6$ , where $n$ is an integer. The only such integers less than 100 are $1^6 = 1$ and $2^6 = 64.$ A) 1 B) 2 C) 3 D) 64
18.	D	18. Parallel lines have equal slopes. The number 2 cannot be written as a product of three identical integers. A) $-1 \times -1 \times -1$ B) $0 \times 0 \times 0$ C) $1 \times 1 \times 1$ D) 2
19.	C	19. If $y = 2x+5$ , then $3y = 6x+15$ ; so $a = 6, b = 15$ , and $a+b = 21.$ A) 7 B) 13 C) 21 D) 30
20.	B	20. My age in years is a two-digit number. Reversing the digits of my age results in my age 18 years ago. So, $10u+t = 10t+u-18.$ Thus, $9t-9u = 18$ or $t-u = 2.$ A) 1 B) 2 C) 3 D) 4
21.	A	21. $\sqrt{M \times A \times T} \times M \times A \times T = M \times A \times T$ , so $H$ must be $M \times A \times T.$ A) $M \times A \times T$ B) $\sqrt{M \times A \times T}$ C) $M^2 \times A^2 \times T^2$ D) 1

