ANSWERS: DIVISION M

SET #13

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 1,000,000	**2A.** 50	**3A.** 4	**4A.** 7	**5A.** 8
1B. 21	**2B.** 10	**3B.** 4	**4B.** 85	**5B.** $\frac{1}{45}$
1C. 15	**2C.** 108	**3C.** $\frac{12}{125}$	**4C.** 36	**5C.** 14
1D. 11	**2D.** 325	**3D.** 72	**4D.** 2	**5D.** 38
1E. 74	**2E.** $840	**3E.** a=6, b=2, c=3	**4E.** 25 sq units	**5E.** 300

SET #14

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 92	**2A.** .826	**3A.** 41	**4A.** 7	**5A.** 36
1B. 4	**2B.** 5	**3B.** 12	**4B.** $\frac{1}{3}$	**5B.** 23
1C. 39 sq mm	**2C.** 47	**3C.** $\frac{3}{5}$	**4C.** 9	**5C.** 56
1D. 5	**2D.** 3	**3D.** 82 units	**4D.** 27π sq cm	**5D.** 363
1E. $\frac{3}{8}$	**2E.** 25	**3E.** 108	**4E.** 48	**5E.** 7

SET #15

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 16	**2A.** 37, 73	**3A.** 11	**4A.** 4	**5A.** 9919
1B. 12	**2B.** 8	**3B.** 7	**4B.** $\frac{5}{7}$	**5B.** 47
1C. 720 sq mm	**2C.** 1000	**3C.** 19	**4C.** 12	**5C.** 7
1D. $125	**2D.** 165	**3D.** (3,10)	**4D.** 10 cm	**5D.** 54
1E. 5	**2E.** 11	**3E.** 43	**4E.** $\frac{3}{36}$	**5E.** 10π

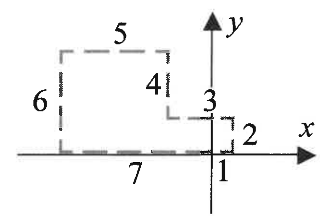
SET #16

Olympiad 1	Olympiad 2	Olympiad 3	Olympiad 4	Olympiad 5
1A. 12	**2A.** Wednesday	**3A.** X=9, Y=3	**4A.** 21	**5A.** 100 miles
1B. 15	**2B.** 32 sq cm	**3B.** 10	**4B.** 10	**5B.** 2
1C. 1 & −11	**2C.** 61	**3C.** $\frac{2}{7}$	**4C.** 19,354	**5C.** −15
1D. 49	**2D.** 5	**3D.** 153	**4D.** 77	**5D.** 66
1E. 114	**2E.** $\frac{24}{5}$	**3E.** 4.8	**4E.** 126	**5E.** 19

SET 14 SOLUTIONS

5D *Strategy: Determine how many of the first 300 numbers can't be used.*
In each of the sets 1 through 100, 101 through 200, and 201 through 300, 10 numbers have a ones digit of 9, and 10 numbers have a tens digit of 9. However, in each set the number ending in 99 has been counted twice, so each set has 19 numbers that contain a digit of 9. In the overall list 1 through 300, 57 numbers must be eliminated. Refill the list with the next 57 numbers, 301 through 357. There are 5 numbers in this set that can't be used (309, 319, 329, 339, 349). Add on 5 more numbers, 358 through 362. One of these, 359, can't be used, so add on one more number. **The 300th number on Sara's list is 363.**

5E *Strategy: Draw possible paths.*
By drawing some paths, you may see two things: (1) the path can turn either left or right, and (2) each horizontal segment has an odd length while each vertical segment has an even length.



Consider first the horizontal segments that can end back at the starting point. Neither {1 unit and 3 units} nor {1, 3 and 5 units} can end at the origin, but {1, 3, 5, and 7 units} can. By traveling 1 and 7 units to the right, and 3 and 5 units to the left, the horizontal part of the path can end at zero.

The longest vertical segment is either 6 or 8 units, the even numbers on either side of 7. Since 2 + 4 = 6, if 2 and 4 are directed up and 6 is directed down, the vertical part of the path also ends at zero. **Thus the shortest possible path consists of 7 segments** and is shown above.

Follow-Ups: *(1) A path consisting of* N *line segments is drawn in the coordinate plane. The first segment starts at (0,0) and is drawn to (2,0). The second segment starts at (2,0) and is drawn to (2,4). Each of the* N *segments is drawn at a right angle to the segment before it and is 2 units longer than that segment. The* N*th segment ends at (0,0). What is the least possible value of* N*?* [7] *(2) What is the least possible value of* N *greater than 7?* [8] *(3) What are the next 2 possible values of* N*?* [15, 16]

Set 15 Olympiad 1

1A METHOD 1: *Strategy: Factor and regroup.*

$$2 \times 6 \times 10 \times 14 = (1 \times 3 \times 5 \times 7) \times (2 \times 2 \times 2 \times 2)$$
$$= (1 \times 3 \times 5 \times 7) \times N$$

Then $N = 2 \times 2 \times 2 \times 2$, so $\boldsymbol{N = 16}$.

METHOD 2: *Strategy: Do the multiplication and solve the equation.*
Since $2 \times 6 \times 10 \times 14 = 1680$ and $1 \times 3 \times 5 \times 7 = 105$, the equation simplifies to $1680 = 105 \times N$. Because $1680 \div 105 = 16$, $N = 16$.

> *FOLLOW-UP: Find the whole number N if $2^2 \times 3^3 \times 4^4 = 27 \times 2^N$.* [10]

1B **METHOD 1:** *Strategy: Start with 1 person in each room.*
After placing 26 people, 1 per room, there are 14 people left over. Place these 14 people, 1 per room, so that there are 14 rooms with 2 people in each. That leaves **12 rooms occupied by exactly one person.**

METHOD 2: *Strategy: Start with 2 people in each room.*
Place the 40 people 2 per room. This fills 20 rooms, leaving 6 rooms empty. Take 1 person from each of 6 of the full rooms to occupy the empty rooms. There are then $6 + 6 = 12$ rooms each occupied by exactly one person.

METHOD 3: *Strategy: Suppose every room has 2 people.*
To fill all 26 rooms with 2 people in each room would require 52 people, There are only 40 people, so $52 - 40 = 12$ rooms will be occupied by just 1 person.

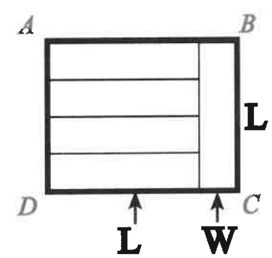
> *FOLLOW-UPS: Suppose 60 people occupy the 26 rooms. There are 1, 2, or 3 people per room. (1) What is the least number of rooms with 3 people?* [8] *(2) What is the greatest number of rooms with 3 people?* [17]

1C *Strategy: Find the dimensions of the small rectangles.*
Represent BC by L and DC by $L + W$.

Then $DC - BC = W$.

$DC - BC = 6$, so $W = 6$ mm and $AD = 4W = 24$ mm.

The area of *ABCD* is $24 \times (24 + 6) =$ **720 sq mm.**



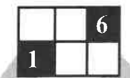
> *FOLLOW-UP: Determine the difference in the lengths BC and CD given that the area of rectangle ABCD is 2000 square mm. [10 mm]*

1D *Strategy: Find the fractional part of the total each paid.*
Call the roommates A, B, and C. A pays half the amount that B and C together pay. That is, of every \$3 paid, A pays \$1 and B and C together pay \$2. A then pays $\frac{1}{3}$ of the total, or \$100. In a similar manner, since B pays $\frac{1}{3}$ the amount paid by A and C together, B pays $\frac{1}{4}$ of the total, or \$75. A and B together pay \$175, so **the third roommate *C* pays** $300 - 175 =$ **\$125.**

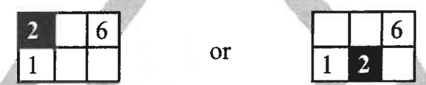
SET 15 SOLUTIONS

1E <u>Strategy</u>: *List the arrangements systematically.*
Denote the 6 girls, from shortest to tallest, by 1, 2, 3, 4, 5, and 6.

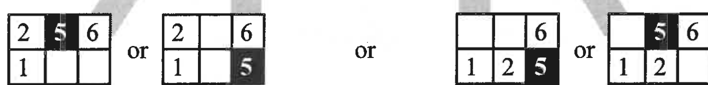
METHOD 1: <u>Strategy</u>: *Position the girls, working from both ends of the list.*
1 must be in the front row at the far left. 6 must be in the back row at the far right.



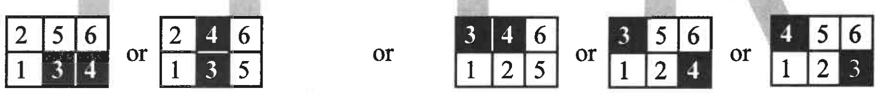
2 can be in either row but must be at the far left of the available spaces.



5 can be in either row but must be at the far right of the available spaces.



3 and 4 go in the remaining spaces. If they are in the same row, 3 goes to the left. If one is directly behind the other, 3 goes in front. Otherwise, 3 and 4 may go in either available space.



In all, 5 arrangements are possible.

METHOD 2: <u>Strategy</u>: *Fill in the front row first.*
As above, 1 must be at the far left of the front row and 6 at the far right of the back row. It seems that the remaining 2 places in the front row can be filled in 6 ways by choosing any two of 2, 3, 4, and 5, namely 2 & 3, 2 & 4, 2 & 5, 3 & 4, 3 & 5, or 4 & 5. However, if 4 and 5 are both in the front row, the back row is 236, with 3 behind 4, which is not allowed by the given conditions. The remaining 5 choices lead to the 5 arrangements shown in Method 1. Five arrangements are possible.

> ***Follow-Up:*** *Suppose 8 girls of different heights are lined up subject to the same conditions. How many arrangements are possible?* [14]

Olympiad 2

2A <u>Strategy</u>: *Consider the ones digits.*
P × Q ends in a 1. The only different single-digit numbers whose product ends in 1 are 3 and 7. Therefore **PQ and QP are 37 and 73** (in either order).

> ***Follow-Ups:*** *(1) Find a single digit number Q, such that P × Q + P + Q = PQ, where PQ is a two-digit number.* [9] *(2) Why does this work?*

SET 15 SOLUTIONS

2B *Strategy: First determine the value of A; then use a common denominator.*
Because $\frac{4}{6}$ is in lowest terms, A must be 1 or 5. But $\frac{5}{6} > \frac{3}{4}$, so $A = 1$.
Then $\frac{2}{12} + \frac{B}{12} = \frac{9}{12}$ and $B = 7$. **The sum of A and B is** $1 + 7 =$ **8.**

2C **METHOD 1:** *Strategy: Look for a pattern in the sum of each row.*
Add the numbers in each row.

Row number	1	2	3	4	...	10
Sum of numbers	1	8	27	64	...	?

1
3 5
7 9 11
13 15 17 19
and so on ...

The sum of the numbers in each row is the cube of that row number.
Then **the sum of the numbers in the 10^{th} row is** $10^3 =$ **1000.**

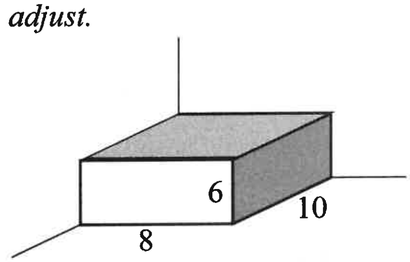
METHOD 2: *Strategy: Find a pattern in each row's first number.*

Row number	1	2	3	4	5
1^{st} entry in row	1	$3 = 2\times1 + 1$	$7 = 3\times2 + 1$	$13 = 4\times3 + 1$	$21 = 5\times4 + 1$

Based on this pattern, the first number in the tenth row is $10\times9 + 1 = 91$. The sum of the ten numbers in the tenth row is $91 + 93 + 95 + ... + 107 + 109 = (91+109) + (93+107) + (95+105) + (97+103) + (99+101) = 5 \times 200 = 1000$.

FOLLOW-UPS: (1) Can you find a pattern in the last number of each row? (2) How is the average (mean) of each row related to the number of that row? (3) How could FOLLOW-UP 2 be used to find the answer to contest problem 2C?

2D **METHOD 1:** *Strategy: Count the cubes on each face; then adjust.*
The greatest number of visible faces is 3. Count the number of cubes visible on each of the 3 faces of the solid. There are $(6 \times 8) + (6 \times 10) + (8 \times 10) = 188$ of them. However, this counts cubes along the 3 visible edges twice and the corner cube 3 times. Count the number of cubes along the edges. There are $10 + 8 + 6$ of them, but the corner cube is counted 3 times here as well, once on each edge. Then $188 - 24$ counts all visible cubes except the corner one.



The greatest number of cubes you can see is $188 - 24 + 1 =$ **165.**

SET 15 SOLUTIONS

METHOD 2: <u>Strategy</u>: *Count and remove the cubes on each visible face.*
As above, the greatest number of visible faces is three. Remove the 80 cubes on the top face to be left with a **<u>5</u>** × 8 × 10 solid. Next, remove the 40 cubes on the front face to be left with a 5 × 8 × **<u>9</u>** solid. The 45 cubes on the side face are the only ones remaining from the original visible cubes. Originally, a total of 80 + 40 + 45 = 165 cubes were visible.

> ***Follow-Up:*** *Suppose you painted all the faces of the rectangular solid in the problem red. How many of the one-inch cubes would have 0 faces painted red? 1 face? 2 faces? 3? 4? 5? 6? [192, 208, 72, 8, 0, 0, 0]*

2E **METHOD 1:** <u>Strategy</u>: *Consider terminating decimals in fractional form.*
A one-place decimal can be written as a fraction with a denominator of 10, a two-place decimal can be written with a denominator of 100, three places with a denominator of 1000, and so on. Then any fraction that can be represented by a terminating decimal must be equivalent to one of these fractions. The only prime factors of 10 or 100 or 1000, … , are 2 and 5. So the unit fractions that can be written as terminating decimals are those with denominators that have only 2s and 5s as prime factors.

- Denominators with only 2 as a factor: 2, 4, 8, 16, 32
- Denominators with only 5 as a factor: 5, 25
- Denominators with both 2 and 5 as factors: 10, 20, 40, 50

There are 11 fractions on the list that have decimal representations that terminate.

METHOD 2: <u>Strategy</u>: *Look for a pattern in the fractions with terminating decimals*
For the unit fractions from $\frac{1}{2}$ to $\frac{1}{10}$ inclusive, the only fractions that have terminating decimals are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, and $\frac{1}{10}$. Notice that the only factors in the denominators are 2 or 5. This is a requirement for the decimal to terminate because 2 and 5 are the only prime factors of 10, 100, 100, and so on. The remaining fractions that terminate in the original list are $\frac{1}{16}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{32}$, $\frac{1}{40}$, and $\frac{1}{50}$. This is a total of 11 fractions.

Olympiad 3

3A <u>Strategy</u>: *Write the problem as an addition.*
In the ones place, 3 + 8 ends in 1 so p is 1; 1 is carried.
In the tens place, 1 + r + 6 ends in 4, so r is 7; 1 is carried.
In the hundreds place, 1 + 4 + q = 8, so q = 3.

$$\begin{array}{r} 4\ r\ 3 \\ +\ q\ 6\ 8 \\ \hline 8\ 4\ p \end{array}$$

Then **p + q + r = 11.**

> ***Follow-Up:*** *Find w + y + z, if 4z × y7 = 1w02. (Note: 4z and y7 are 2-digit numbers and 1w02 is a 4-digit number.)* [16]

SET 15 SOLUTIONS

3B *Strategy: Determine possible values of the radicand.*
If $\sqrt{50-x}$ is a positive integer and x is positive, $(50-x)$ must be a perfect square less than 50. So $(50-x)$ can be any of 49, 36, 25, 16, 9, 4, or 1, leading to $x = 1, 14, 25, 34, 41, 46$, or 49, respectively. **There are 7 positive integer values of x for which $\sqrt{50-x}$ is a whole number.**

Follow-Up: *For how many whole number values of x is $\sqrt{108-3x}$ a whole number?* [3]

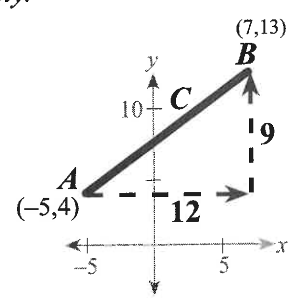
3C *Strategy: List values of the prime, P, in increasing order.*
The first row lists prime numbers. The second row lists the difference between 98 and the prime. The third row states if the entry in the second row is prime and identifies a simple factor if the entry is composite.

P	2	3	5	7	11	13	17	19
$98-P$	96	95	93	91	87	85	81	79
Is $98-P$ prime?	no (÷2)	no (÷5)	no (÷3)	no (÷7)	no (÷3)	no (÷5)	no (÷3)	PRIME

Therefore, **the least value of P is 19.**

Follow-Up: *Find a whole number value for N such that the value of $N^2 + N + 41$ is not prime.* [The least is 40; the most readily found is 41.]

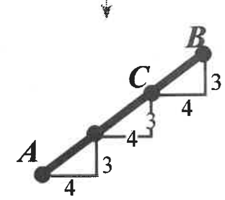
3D METHOD 1: *Strategy: Determine the x- and y-coordinates separately.*
C is $\frac{2}{3}$ of the way from A to B. Then the x-coordinate of C is $\frac{2}{3}$ of the way from the x-coordinate of A to the x-coordinate of B. The x (horizontal) distance from A to B is $7-(-5)=12$ and $\frac{2}{3}$ of $12 = 8$. The x-coordinate of C is $(-5)+8=3$.



Similarly, the y-coordinate of C is $4 + \frac{2}{3} \times (13-4) = 10$.

The coordinates of point C are (3,10).

METHOD 2: *Strategy: Show the $\frac{2}{3}$ visually.*
Replace the horizontal and vertical distances of 12 and 9 in *method 1* by three 4 by 3 "steps" as shown. To go from A to C, start at (–5,4), move to the right 4 and 4 again, and move up 3 and 3 again. Then $(-5+4+4,\ 4+3+3)$ yields (3,10).



Follow-Up: *Suppose point C is $\frac{3}{5}$ of the way from A to B, the coordinates of A are (7,21) and the coordinates of C are (13,9). What are the coordinates of B?* [(17,1)]

SET 15 SOLUTIONS

3E **METHOD 1:** *Strategy: Factor the given expression.*

$7! \;=\; 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$
$\;\;\;\;\;= \;\; 5! \times (6 \times 7).$

Then $5! + 7! = 5! + (5! \times 6 \times 7)$
$= (5! \times 1) + (5! \times 6 \times 7)$
$= 5! \times (1 + 6 \times 7)$ [by the distributive property]
$= (1 \times 2 \times 3 \times 4 \times 5) \times 43.$

43 is larger than any prime factor of 5!.

The largest prime factor of 5! + 7! is 43.

METHOD 2: *Strategy: Perform the indicated operations and then factor.*
$5! + 7! = 120 + 5{,}040 = 5{,}160.$
Factor out as many small primes as you can: $5{,}160 = 2^3 \times 3 \times 5 \times 43$.
The largest prime factor of 5! + 7! is 43.

FOLLOW-UP: *Find the value of N for which $N! \times 4! = (N+1)!$* [23]

Olympiad 4

4A *Strategy: Use reasoning.*
First fill in the boxes labeled A and B which must contain the numbers 3 and 4. A is not 4, so A must be 3 and B must be 4. Next fill in box D with a 2. Then C is 1 or 4. Since C cannot be 4, C is 1 and **the box marked with an X contains a 4.**

	4	1	A
		B	2
3	C	D	*X*
		3	

FOLLOW-UP: *Create your own 4 × 4 Sudoku-type puzzle by filling in the answers and then erasing some of the numbers. Make sure that the numbers you leave allow only 1 solution.*

4B **METHOD 1:** *Strategy: Factor the numerator and denominator.*
Factor out the common factor in both the numerator and denominator and then cancel to get the following.

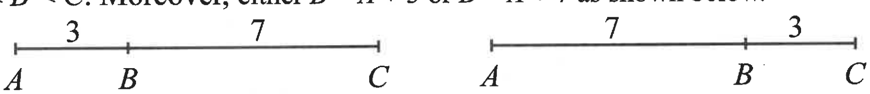
$$\frac{5\,\cancel{(1-2+3-4+\ldots-98+99-100)}}{7\,\cancel{(1-2+3-4+\ldots-98+99-100)}} = \frac{\mathbf{5}}{\mathbf{7}}$$

METHOD 2: *Strategy: Pair numbers in both numerator and denominator.*
Numerator: $(5-10)+(15-20)+\ldots+(495-500) = (-5)+(-5)+(-5)+\ldots+(-5) = -250$
Denominator: $(7-14)+(21-28)+\ldots+(693-700) = (-7)+(-7)+(-7)+\ldots+(-7) = -350$
Then $\frac{-250}{-350} = \frac{5}{7}$

METHOD 3: *Strategy: Look for a pattern in the partial sums.*
Start with $\frac{5}{7}$, next $\frac{5-10}{7-14}$, then $\frac{5-10+15}{7-14+21}$, and so on. In each case the fraction equals $\frac{5}{7}$. In the given fraction both the numerator and denominator contain the same number of terms, 100, so the value remains at $\frac{5}{7}$.

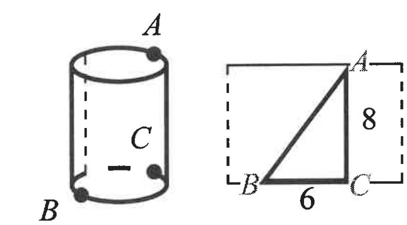
> **FOLLOW-UP:** *Explain why the successive fractions listed in Method 3 are equivalent.*

4C *Strategy: Set up and solve possible equations.*
C is 10 more than A. Also, B. is 3 away from one number and 7 away from the other number. Then $A < B < C$. Moreover, either $B = A + 3$ or $B = A + 7$ as shown below.



Substitute $C = A + 10$ and each possibility for B into $A + B + C = 32$. The result will be two equations: $A + (A + 3) + (A + 10) = 32$ and $A + (A + 7) + (A + 10) = 32$. The first equation results in $A = \frac{19}{3}$ and the second equation results in $A = 5$. A is a whole number so $A = 5$ and $\boldsymbol{B = 12}$.

4D *Strategy: Remove the label from the can.*
Cut open the cylinder along the dotted line and unroll it to get a rectangle with A along the top edge and B along the bottom edge. This makes it easier to see the shortest distance from A to B. Then connect A and B with a straight line segment. Place C as shown to form right triangle ACB. Since C is opposite B on the cylinder, BC is one-half the circumference. Then $AC = 8$ cm and $BC = 6$ cm. Apply the Pythagorean Theorem or recognize the Pythagorean triple 6-8-10 to get $AB = 10$ cm. **The shortest distance from A to B is 10 cm.**



4E *Strategy: List the primes and subtract each prime from 30.*
The probability is the ratio of the number of pairs of primes whose sum is 30 to the total number of all pairs of primes.

Sum is 30: The first nine prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, and 23. Begin with the greatest prime. $30 - 23 = 7$, also a prime. $30 - 19 = 11$, a prime. $30 - 17 = 13$, a prime. The next prime is 13 which is accounted for; There's no need to test further. Only 3 pairs of primes have a sum of 30: {7, 23}, {11, 19}, and {13, 17}.

All pairs of primes: There are several ways to count all 36 possible pairs of primes. Three methods are offered.

SET 15 SOLUTIONS

1. Pair each prime with the greater primes: thus 2 is paired with each of the 8 other primes greater than 2, 3 with each of the 7 primes greater than 3, 5 with each of the 6 primes greater than 5, and so on. In all there are $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$ pairs of primes.
2. There are 9 possible values for the first of the two primes. Pair each of these with any of the 8 remaining primes, a total of $9 \times 8 = 72$. This, however, counts each pair twice (e. g. 2 paired with 3 and 3 paired with 2). The total number of pairs is $72 \div 2 = 36$.
3. List all pairs of primes in an orderly manner.

The probability that the sum of the 2 primes 30 is $\frac{3}{36} = \frac{1}{12}$.

> *FOLLOW-UPS: (1) Two different primes are selected from the first 9 prime numbers. What is the probability that their sum is odd?* [$\frac{16}{72}$ or $\frac{2}{9}$] *(2) Two primes, not necessarily different, are selected from the first 9 prime numbers. What is the probability that their sum is odd?* [$\frac{16}{81}$] *(3) Two different primes are selected from the first 20 prime numbers. What is the probability that their sum is 30?* [$\frac{3}{190}$]

Olympiad 5

5A *Strategy: Maximize the digits, working from the left.*
The largest possible value for A is 9. Since A = B × C, the values of B and C are either 3 and 3, or 9 and 1. Of these, the largest possible value of B is 9. Then C is 1. Thus B = C × D becomes 9 = 1 × D and D is 9. **The greatest 4-digit number ABCD is 9919.**

> *FOLLOW-UP: Find the largest 4-digit number, ABCD, so that A × B = C and A × C = D.* [3139]

5B **METHOD 1:** *Strategy: Use algebra and the definition of mean.*
Rewrite $23 - x = y - 71$ as $x + y = 23 + 71 = 94$.

The mean of x and y is $\frac{1}{2}(x+y) = \frac{1}{2}(94) = 47$.

METHOD 2: *Strategy: Assign values to x .*
The wording of the question implies that there is a single answer no matter what value is assigned for x. Therefore assign any value to find the mean: suppose $x = 1$. Then $y = 93$ and the mean is $\frac{1}{2}(1 + 93) = 47$.

To check, assign at least two very different values to $23 - x = y - 71$: If $x = 80$, then $y = 14$ and again the mean is 47. And if $x = -7$, then $y = 101$ and still the mean is 47.

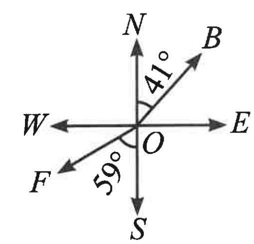
SET 15 SOLUTIONS

5C *Strategy: Find a pattern in the successive powers of 2 and of 3.*
$2^1 = \underline{2}$, $2^2 = \underline{4}$, $2^3 = \underline{8}$, $2^4 = 1\underline{6}$, $2^5 = 3\underline{2}$, $2^6 = 6\underline{4}$, $2^7 = 12\underline{8}$, $2^8 = 25\underline{6}$, and so on. The ones digits repeat in the pattern 2, 4, 8, 6 and then 2, 4, 8, 6, and so on. Then 2^4, 2^8, 2^{12}, and 2^{16} all have the same ones digit, 6. Similarly, 2^3, 2^7, 2^{11}, and 2^{15} all have the same ones digit, 8.

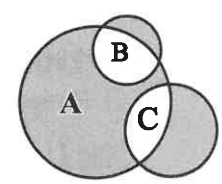
Repeat the process on powers of 3. The successive ones digits are 3, 9, 7, 1 and then 3, 9, 7, 1 again, and so on. Then 3^{10} has the same as the ones digit as 3^2, namely 9. Thus **the ones digit in $2^{15} + 3^{10}$** is the same as that of 8 + 9, which **is 7.**

> **FOLLOW-UPS:** *(1) What is the ones digit in the product of $2^{2012} \times 3^{2013} \times 5^{2014}$?* [0] *(2) How many consecutive zeros appear at the end of the product?* [2012]

5D *Strategy: Draw a picture.*
The angle through which the boat turns is $\angle BOF$. $m\angle BOE = 90 - 41 = 49$ and $m\angle WOF = 90 - 59 = 31$. To begin at a heading of B and finish at a heading of F, the boat must turn either $41 + 90 + 31 = 162°$ counterclockwise, or $49 + 90 + 59 = 198°$ clockwise. The lesser angle requires less time, and at 3° per second, **the least time required is** $162 \div 3 =$ **54 seconds.**



5E *Strategy: Find the total of the unshaded areas.*
The sum of the areas of the 3 circles, $4\pi + 9\pi + 16\pi = 29\pi$, includes each interior region in the picture. However, it includes regions B and C twice, since each is part of two circles. The sum of the areas of B and C is then $(29\pi - 17\pi) \div 2 = 6\pi$. The largest circle, whose area is 16π, consists of regions A, B, and C, so **the area of region A alone is** $16\pi - 6\pi =$ **10π.**



Set 16 Olympiad 1

1A METHOD 1: *Strategy: Use the distributive property.*
$2013 \times 10{,}001 = 2013 \times (10{,}000 + 1) = 20130000 + 2013$. There are no carries in the addition of the two addends, so the sum of the digits in the sum is the same as the sum of the digits in the two numbers. **The sum of the digits in the product is** 6 + 6 = **12.**

METHOD 2: *Strategy: Do the multiplication.*
$2013 \times 10{,}001 = 20{,}132{,}013$. The sum of the digits in the product is 12.