23. Use the Pythagorean Theorem. In 1 hour at these rates, Fred would be c km from his starting point where $c^2 = 1^2 + (3/4)^2$. Simplifying, $c^2 = 25/16$ and $c = 5/4$. In 1 min. Fred would travel $(5/4 \div 60)$ km.

A) $\frac{1}{100}$ B) $\frac{1}{70}$ C) $\frac{1}{48}$ D) $\frac{1}{20}$

23. C

24. Since $100^{100} = (100^{50})^2$, it follows that $\sqrt{100^{100}} = 100^{50}$.

A) 10^{10} B) 10^{50} C) 100^{10} D) 100^{50}

24. D

25. $(x+1)^4 = (x+1)^2(x+1)^2 = (x^2+2x+1)^2 = x^4 + 4x^3 + 6x^2 + 4x + 1$. The desired sum is $4 + 4 = 8$.

A) 4 B) 6 C) 8 D) 10

25. C

26. $(x+4)^2 = (y+5)^2$, so $x^2 + 8x + 16 = y + 10y + 25$. Thus, $x^2 + 8x = y^2 + 10y + 9$.

A) $y^2 + 10y + 9$ B) $y^2 + 9y + 10$ C) $y^2 + 10y$ D) $y^2 + 9y$

26. A

27. $12^{1200} = (3^{1200})(4^{1200}) = (3^{1200})(2^2)^{1200} = (3^{1200})(2^{2400})$, so $x = 2400$.

A) 600 B) 1200 C) 2400 D) 3600

27. C

28. Let $3f$ = # of grandfathers and $4m$ = # of grandmothers. If $\frac{2}{3}$ of the grandfathers and $\frac{1}{2}$ of the grandmothers bought coats, then $2f$ grandfathers bought coats and $2m$ grandmothers bought coats. We know that $3f + 4m = 100$ and $2f = 3m + 10$. Solving, $f = 20$ and $m = 10$. Thus, # of grandfathers who bought coats = $2f = 40$.

A) 35 B) 40 C) 55 D) 60

28. B

29. If $r = (2 + 4 + 6 + \ldots + 2010)$ and $s = (1 + 3 + 5 + \ldots + 2009)$, then $r - s = (2-1) + (4-3) + (6-5) + \ldots + (2010-2009) = 1 + 1 + 1 + \ldots + 1 = 1005$.

A) 1005 B) 1006 C) 2010 D) 2011

29. A

30. Clearing fractions, $(y^2 - x^2) + (y - x) = 0$. Factoring, $(y-x)(y+x) + (y-x) = 0$. Dividing by $(y-x)$, which is not 0, $y + x + 1 = 0$, so $x + y = -1$.

A) -2 B) -1 C) 0 D) 1

30. B

The end of the contest **A**

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Information & Solutions

Spring, 2010

Contest Information

A

- **Solutions** Turn the page for detailed contest solutions (written in the question boxes) and letter answers (written in the *Answer Column* to the right of each question).
- **Scores** Please remember that *this is a contest, and not a test*—there is no "passing" or "failing" score. Few students score as high as 24 points (80% correct); students with half that, 12 points, *deserve commendation!*
- **Answers and Rating Scales** Turn to page 151 for the letter answers to each question and the rating scale for this contest.

1. If $b = 1$, $e = 2b = 2$, $a = 3e = 6$, and $r = 4a = 24$, then $b + e + a + r = 1 + 2 + 6 + 24 = 33$.
A) 4 B) 10 C) 24 D) 33 — **1. D**

2. When $x = 1$, each power of x is also 1. Therefore, the values of the choices are 0, -1, -2, and -3.
A) $x - 1$ B) $x^2 - 2$ C) $x^3 - 3$ D) $x^4 - 4$ — **2. A**

3. $(-4)^2(-3)^0(-2)^1(-1)^0 = (16)(1)(-2)(1) = -32$.
A) -32 B) 0 C) 16 D) 32 — **3. A**

4. $x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$.
A) $x + 4$ B) $x - 4$ C) $x + 2$ D) $x - 1$ — **4. C**

5. x^{-2010} is the reciprocal of x^{2010}. For positive values of x^{-2010}, x^{-2010} is greatest when x^{2010} is least, and x^{2010} is least when $x = 100$.
A) 100 B) 200 C) 300 D) 400 — **5. A**

6. If $300x = 450 - 300y$, then $300x + 300y = 450$ and $x + y = 1.5$.
A) -3 B) 1.5 C) 15 D) 30 — **6. B**

7. $(2x^4 + 4x^2) + (3x^4 - 5x^2) - (4x^4 - 6x^2) = (2x^4 + 3x^4 - 4x^4) + (4x^2 - 5x^2 + 6x^2)$.
A) $x^4 - 7x^2$ B) $x^4 + 5x^2$ C) $x^4 - 3x^2$ D) $x^4 + x^2$ — **7. B**

8. If $x > 0$, then the additive inverse of x divided by the reciprocal of x equals $-x$ divided by $\frac{1}{x} = (-x)(x) = -x^2$.
A) the square of x B) the reciprocal of the square of x
C) the square root of x D) the additive inverse of the square of x — **8. D**

9. $(a^4)^3 = a^{12}$ and $(a^{12})^2 = a^{24}$.
A) a^9 B) a^{24} C) a^{36} D) a^{64} — **9. B**

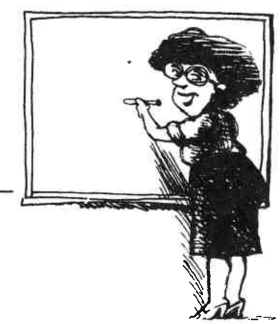
10. If $g - c = 2$ and $(g - c)(g + c) = 20$, then $2(g + c) = 20$ and $g + c = 10$. Thus, $g = 6$ and $c = 4$.
A) 6 B) 8 C) 10 D) 12 — **10. A**

11. $\frac{x}{100} \times \frac{y}{100} \times 100\,000 = 10xy$.
A) $x + y$ B) xy C) $\frac{x+y}{10}$ D) $10xy$ — **11. D**

12. $\sqrt{2} + \sqrt{4} + \sqrt{8} + \sqrt{16} = \sqrt{2} + 2 + 2\sqrt{2} + 4 = 6 + 3\sqrt{2}$.
A) $\sqrt{30}$ B) $6 + 3\sqrt{2}$ C) $10\sqrt{2}$ D) $3 + 4\sqrt{2}$ — **12. B**

Go on to the next page A

13. The board's area is ℓw and its perimeter is $2\ell + 2w$. Adding its area to its perimeter and subtracting twice its length, I get $\ell w + 2w$. Dividing by w, I get $\ell + 2$. Finally, subtracting 2, I get ℓ.
A) length B) width C) diagonal D) area — **13. A**



14. If the equation of ℓ is $39x + 54y = 101$, its slope is $-\frac{39}{54}$. Choice C has slope $\frac{54}{39}$.
A) $54x + 39y = 16$ B) $36x + 26y = 56$
C) $54x - 39y = 1$ D) $39x - 54y = 101$ — **14. C**

15. $\frac{x-1}{x-2} \times \frac{x-2}{x-3} \times \frac{x-3}{x-4} \times \frac{x-4}{x-5} = \frac{x-1}{1} \times \frac{1}{x-5} = \frac{x-1}{x-5}$.
A) x^4 B) $(x-1)^4$ C) $(x-5)^{-4}$ D) $\frac{x-1}{x-5}$ — **15. D**

16. If $b = 4$, then $x^2 + 4x + 4 = 0$. Factoring, $(x + 2)(x + 2) = 0$, so $x = -2$ or -2.
A) -6 B) -2 C) 0 D) 4 — **16. D**

17. If $\frac{x}{y} = \frac{2}{9}$ and $\frac{y}{z} = \frac{5}{4}$, then $\frac{x}{y} \times \frac{y}{z} = \frac{x}{z} = \frac{2}{9} \times \frac{5}{4} = \frac{10}{36} = \frac{5}{18}$.
A) $\frac{5}{18}$ B) $\frac{8}{45}$ C) $\frac{6}{14}$ D) $\frac{7}{13}$ — **17. A**

18. $(x^3 - 4)^2 + 8x^3 - 25 = (x^6 - 8x^3 + 16) + 8x^3 - 25 = x^6 - 9 = (x^3 + 3)(x^3 - 3)$.
A) $x^3 + 5$ B) $x^3 + 4$ C) $x^3 + 3$ D) $x^3 + 2$ — **18. C**

19. Since $z = (x + y)/2$, $x + y = 2z$. Thus, $x - y = (x + y) - 2y = 2z - 2y = 2(z - y)$. Therefore, $x - y$ is always an even integer.
A) 0 B) 1 C) y D) z — **19. B**

20. If 24 workers can build a house in 10 hours, then 1 worker could build the house in 240 hours, and 40 could build the house in 6 hours.
A) $\frac{50}{3}$ B) 6 C) 14 D) $\frac{64}{10}$ — **20. B**

21. If $(x + 4)^2 = 5^{12}$ and $x > 0$, then $x + 4 = 5^6$.
A) $5^6 - 4$ B) 5^6 C) $5^6 + 4$ D) $5^6 + 8$ — **21. A**

22. If x is a prime greater than 3, then the positive divisors of $6x$ are 1, 2, 3, 6, x, $2x$, $3x$, and $6x$.
A) 2 B) 4 C) 6 D) 8 — **22. D**

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