ANSWER KEYS TO TEST 4

SPRINT ROUND

1. 768.
2. 100.
3. 12.
4. 32.
5. 1.331 km.
6. 5/2.
7. 20.
8. 32.
9. 5050.
10. 60.
11. 14.
12. 4,665.
13. $3-2\sqrt{2}$.
14. 12.
15. 2016π.
16. 1/21.
17. 6.
18. $2+2\sqrt{2}$.
19. 13 days.
20. 12 cm.
21. 30.
22. 2.
23. $30+\sqrt{73}$.
24. $\frac{17}{450}$.
25. 2 hours.
26. 24.
27. 473.
28. 960π.
29. 396.
30. 6.

TARGET ROUND

1. 64.
2. 21 km.
3. 43.
4. 4788.
5. 756.
6. 214.
7. 44.
8. $\frac{3}{2}$.

SPRINT ROUND SOLUTIONS

1. **Solution:** 768.
Todd sold 1/3 of 2016 bards. The number of cards left is
$2016 - \frac{1}{3} \times 2016 = 2016 \times \frac{2}{3} = 1344$.

Todd sold 3/7 of 1344 bards. The number of cards left is
$1344 - \frac{3}{7} \times 1344 = 1344 \times \frac{4}{7} = 768$.

2. **Solution:** 100.
Method 1:
The fractional part of the number of marbles that are neither green nor yellow is
$1 - \frac{35}{100} - \frac{20}{100} = \frac{45}{100}$.

Since 80% of the remaining ones are blue, the fractional part of the number of blue is
$\frac{80}{100} \times \frac{45}{100} = \frac{36}{100}$.

We know that the number of blue marbles is 36. So 36 ÷ 36% = 100.

Method 2:
Let x be the number of marbles in the bag.

The number of blue marbles is $\frac{80}{100} \times (1 - \frac{35}{100} - \frac{20}{100})x$ and
$\frac{80}{100} \times (1 - \frac{35}{100} - \frac{20}{100})x = 36 \Rightarrow \quad x = 100$.

3. **Solution:** 12.
The amount of time and the amount of wheat do not matter.
To find the average number of eggs produced per hen, we simply divide the number of eggs by the number of hens.
888 ÷ 74 = 12.

4. **Solution:** 32.
Container 10 has 2^{10} marbles and container 9 has 2^9 marbles. So container 5 has $2^5 = 32$ marbles.

5. **Solution:** 1.331 km.
$d = 11t^2 = 11 \times 11^2 = 11 \times 121 = 1331$ m = 1.331 km.

6. **Solution:** 5/2.

Method 1:
$24 = 24 \times 1 = 12 \times 2 = 8 \times 3 = 6 \times 4$.

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{12}+\frac{1}{24}=\frac{24+12+8+6+4+3+2+1}{24}=\frac{60}{24}=\frac{5}{2}.$$

Method 2:
The answer is $\sigma(n)/n$, where $\sigma(n)$ is the sum of the factors of n.

$$\sigma(n)=(p_1^{\,a}+p_1^{\,a-1}+\ldots+p_1^{\,0})(p_2^{\,b}+p_2^{\,b-1}+\ldots+p_2^{\,0})\ldots(p_k^{\,m}+p_k^{\,m-1}+\ldots+p_k^{\,0})$$

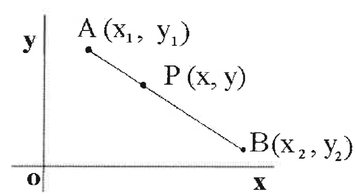
$24 = 3 \times 8 = 3 \times 2^3$

The sum of all divisors:

$$\sigma(n)=(p_1^{\,a}+p_1^{\,a-1}+\ldots+p_1^{\,0})(p_2^{\,b}+p_2^{\,b-1}+\ldots+p_2^{\,0})$$
$$=(3^1+3^0)(2^3+2^2+2^1+2^0)=(3+1)(8+4+2+1)=4\times 15=60$$

The answer is 60/24 = 5/2.

7. **Solution:** 20.
Let point P be (x, y). $\lambda = AP : PB = 2 : 3$.



$$x=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{4+\frac{2}{3}\times 20}{1+\frac{2}{3}}=\frac{52}{5}$$

and $$y=\frac{y_1+\lambda y_2}{1+\lambda}=\frac{12+\frac{2}{3}\times 6}{1+\frac{2}{3}}=\frac{48}{5}.\Rightarrow x+y=\frac{52}{5}+\frac{48}{5}=20.$$

8. **Solution:** 32.
We see that $30^3 = 27,000 < 30,000 < 40^3 = 64,000$.
So the number is larger than 30. We try $31^3 = 961 \times 31 = 29,791$.
So the answer is 31 + 1 = 32. We check and know that 32 work: $32^3 = 1024 \times 32 = 32,768 > 30,000$.

9. **Solution:** 5,050.

$$1 + 2 + 3 + \ldots + 100 = \frac{(1+100)\times 100}{2} = 5050.$$

10. **Solution:** 60.
Method 1:
Let x be the number of balls in the bag after addition.

$$(x - 15 - 3)\times\frac{1}{2} = \frac{3}{5}x - 15 \quad \Rightarrow \quad x = 60.$$

Method 2:
Let x be the number of balls originally in the bag before addition.
Let r be the number of red balls originally in the bag before addition.

Originally: $\frac{r}{x} = \frac{1}{2} \quad \Rightarrow \quad r = \frac{1}{2}x \quad \Rightarrow \quad 5r = \frac{5}{2}x$ (1)

After addition: $\frac{r+15}{x+15+3} = \frac{3}{5} \Rightarrow 5r + 75 = 3x + 45 + 9 \Rightarrow 5r + 21 = 3x$ (2)

Substituting (1) into (2): $\frac{5}{2}x + 21 = 3x \quad \Rightarrow \quad \frac{1}{2}x = 21 \quad \Rightarrow \quad x = 42$

So the answer is 42 + 15 + 3 = 42 + 18 = 60.

11. **Solution:** 14.
$x \# y = xy - x - y + 11 = 21 \quad \Rightarrow (x-1)(y-1) + 10 = 21$
$\Rightarrow (x-1)(y-1) = 11$.
Since both x and y are positive integers, we have
$x - 1 = 1$
$y - 1 = 11$.
or
$x - 1 = 11$
$y - 1 = 1$.

In both case, $x + y = 14$.

12. **Solution:** 4665.

$1 + 2 + 3 + \ldots + 100 = \frac{(1+100)\times 100}{2} = 5050$.

$1^2 + 2^2 + 3^2 + \ldots + 10^2 = \frac{n(n+1)(2n+1)}{6}$.

$= \frac{10\times 11\times 21}{6} = \frac{2\times 5\times 11\times 3\times 7}{6} = 5\times 11\times 7 = 35\times 11 = 385$.

The answer is $5050 - 385 = 4665$.

13. **Solution:** $3 - 2\sqrt{2}$.

$x + \frac{1}{x} = 6 \quad \Rightarrow \quad x^2 + 1 = 6x \quad \Rightarrow \quad x^2 - 6x + 1 = 0$.

By the quadratic formula, $x_{1,2} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4\times 1\times 1}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$.

Since x is the smallest, $x = 3 - 2\sqrt{2}$.

14. **Solution:** 12.

Let g be the number of green balls and b be the number of blue balls. y be the number of balls Alex takes out.

$$b + g = 25 \qquad (1)$$

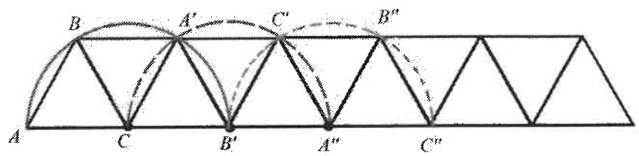
$$y = \frac{b}{2} + \frac{g}{3} = \frac{b}{2} + \frac{25-b}{3} = \frac{b}{2} + \frac{25}{3} - \frac{b}{3} = \frac{b+50}{6}.$$

Since we want the greatest value of y, we let $b = 22$ to get $y = 12$.

15. **Solution:** 2016π.

As shown in the figure below, point A will touch the base line again for every 3 turns ($A - A' - A''$). The distance it travels for every 3 turns is $\frac{2\pi r}{6}\times 4 = \frac{2\pi\times 6}{6}\times 4 = 8\pi$.

So after 756 turns, the distance travelled by point A is x and $\frac{8\pi}{3} = \frac{x}{756} \quad \Rightarrow x = 2016\pi$



16. **Solution:** 1/21.

There are $\left\lfloor \frac{2016}{21} \right\rfloor = 96$ positive integer less than or equal to 2016 that are divisible by 21, where $\lfloor x \rfloor$ is called the floor function.

$P = \frac{96}{2016} = \frac{96}{21 \times 96} = \frac{1}{21}$.

17. **Solution:** 6.

Let *x, y,* and *z* be the numbers of 30 acres lots, 15 acres lots, and 5 acres lots, respectively.

We have the following equation: $30x + 15y + 5z = 100 \quad \Rightarrow 6x + 3y + z = 20$.

Since the developer must have at least one lot of each type, $x, y, z > 0$.

x can only be 2, or 1.

When $x = 2$, $6x + 3y + z = 20 \quad \Rightarrow \quad 3y + z = 8$.

If $y = 1$, $z = 5$; if $y = 2$, $z = 2$. (Two ways).

When $x = 1$, $6x + 3y + z = 20 \quad \Rightarrow \quad 3y + z = 14$.

If $y = 1$, $z = 11$; if $y = 2$, $z = 8$; $y = 3$, $z = 5$; if $y = 4$, $z = 2$. (Four ways).

The solution is: 6 ways.

18. **Solution:** $2 + 2\sqrt{2}$.

By the Pythagorean Theorem, we have $a^2 + b^2 = c^2 \Rightarrow \quad a^2 + b^2 = 4 \quad (1)$

We are given that $\frac{1}{2}ab = 1 \quad \Rightarrow \quad 2ab = 4 \quad (2)$

(1) + (2): $(a + b)^2 = 8 \Rightarrow \quad a + b = 2\sqrt{2}$.

The perimeter is $2 + 2\sqrt{2}$.

19. **Solution:** 13 days.
Let d be the constant eating rate of each duck and c be the constant rate for each chicken. n be the number of days it lasts when feeding only 8 chickens.

$$6(\frac{16}{d}+\frac{12}{c})=1 \quad \Rightarrow \quad \frac{96}{d}+\frac{72}{c}=1 \qquad (1)$$

$$8(\frac{9}{d}+\frac{10}{c})=1 \quad \Rightarrow \quad \frac{72}{d}+\frac{80}{c}=1 \qquad (2)$$

$$(1)-(2): \frac{24}{d}=\frac{8}{c} \quad \Rightarrow \quad d=3c \qquad (3)$$

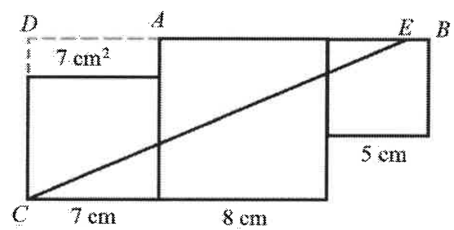
Substituting (3) into (2): $\frac{72}{3c}+\frac{80}{c}=1 \Rightarrow \frac{72+240}{3c}=1 \Rightarrow c=104$

$$n\times\frac{8}{104}=1 \quad \Rightarrow n=13.$$

20. **Solution:** 12 cm.
Extend BA to meet CD at D.
The area of the original figure is $7^2 + 8^2 + 5^2 =$ 138 cm^2.
The area of the triangle CDE is

$$\frac{DE\times CD}{2}=\frac{138}{2}+7 \quad \Rightarrow \quad \frac{(7+AE)\times 8}{2}=76$$

$\Rightarrow \quad AE=12$.



21. **Solution:** 30.
Let x be the number of marbles that must be added to make the probability of selecting a blue marble greater than 2/3.

$$P=\frac{11+x}{7+11+13+x}>\frac{2}{3} \quad \Rightarrow \quad 3(11+x)>2(7+11+13+x) \Rightarrow$$

$$33+3x>62+2x \Rightarrow \quad x>62-33=29.$$

The answer is 29 + 1 = 30.

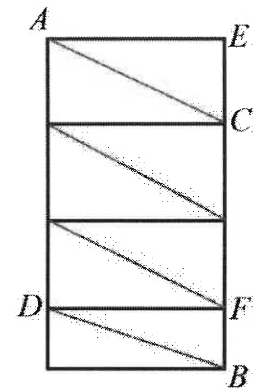
22. **Solution:** 2.
The distance D traveled by the center of circle B can be used as a representative distance traveled by it.

$D = 2\pi(R - r) = 12\pi - 4\pi = 8\pi.$

The number of revolutions is $\dfrac{8\pi}{2\pi \times 2} = 2$

23. **Solution:** $30 + \sqrt{73}$.

If the silo were cut and spread flat, it would form a rectangular shape 8 feet wide and 21 feet high. So the length of the wire is $2AC + BD$ as shown in the figure.



$EC = 21/3.5 = 6$. $AE = \pi \times (8/\pi) = 8$.

$AC = \sqrt{AE^2 + EC^2} = \sqrt{6^2 + 8^2} = 10$.

$BD = \sqrt{DF^2 + FB^2} = \sqrt{8^2 + 3^2} = \sqrt{73}$.

The answer is $30 + \sqrt{73}$.

24. **Solution:** $\dfrac{17}{450}$.

We are given that the number contains one 3 (the hundreds digit).
Let the last two digits be a and b.
We have

$a + b = 0$	(00)
$a + b = 3$	(30, 03, 21, 12)
$a + b = 6$	(60, 06, 51, 15, 42, 24, 33)
$a + b = 9$	(90, 09, 81, 18, 72, 27, 63, 36, 54, 45)
$a + b = 12$	(93, 39, 84, 48, 75, 57, 66)
$a + b = 15$	(96, 69, 87, 78)
$a + b = 18$	(99)

Total: 1 + 4 + 7 + 10 + 7 + 4 + 1 = 34.

The probability is $P = \dfrac{34}{900} = \dfrac{17}{450}$.

25. **Solution:** 2 hours.

Method 1:

Half of the time is $90 \div (9 + 6) = 6$ hours.

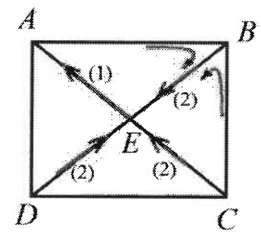
It takes him x hours to ride the first 45 miles, where $x = 45/9 = 5$ hours.

$y = 2 \times 6 - 5 = 7$ hours. The difference is $y - x = 7 - 5 = 2$ hours.

Method 2:
Let m be the distance travelled in first half of the time.
$$\frac{m}{9} = \frac{90-m}{6} \Rightarrow \quad \frac{m}{3} = \frac{90-m}{2} = \frac{m+90-m}{3+2} = 18 \Rightarrow \quad m = 54.$$
The time taken for Alex to ride first 45 miles is 45/9 = 5.

The time taken for Alex to ride the second 45 miles is (54 – 45)/9 + (90 – 54)/6 = 1 + 6 = 7. The difference is $y - x = 7 - 5 = 2$ hours.

26. **Solution:** 24.
We see one way from E to A. We see two ways from each point B, C, or D to E. For example, we see two ways from B to E: ABE, and $ADCBE$ (see figure).
We have 6 ways from A to E.
Similarly, we have 6 ways from each of vertices D and C.
The answer is $6 \times 4 = 24$.



27. **Solution:** 473.
$$0.\overline{3A7} = \frac{3A7}{999}.$$
So $$\frac{B}{1221} = \frac{3A7}{999} \quad \Rightarrow \quad \frac{B}{11} = \frac{3A7}{9} \quad \Rightarrow \quad B = \frac{3A7}{9} \times 11.$$
Since B is a positive integer and 9 and 11 are relatively prime, the 3-digit number $3A7$ must be divisible by 9.
We know that A is a digit, so A must be 8. $B = \frac{387}{9} \times 11 = 43 \times 11 = 473$.

28. **Solution:** 960π.
The minute hand completes one round in every hour. The distance travelled by the tip of the minute hand in 72 hours is $72 \times 2\pi \times 7$.

The hour hand completes one round in every 12 hours. The distance travelled by the tip of the hour hand in 72 hours is $\frac{72}{12} \times 2\pi \times 4$.

The positive difference is $72\times 2\pi\times 7 \;-\; \frac{72}{12}\times 2\pi\times 4 \;=\; 960\pi$.

29. **Solution:** 396.
We know that $D = 1$.

$$\begin{array}{r} ABC \\ +1EF \\ \hline 1GB1 \end{array}$$

We have
Case 1: $A + 1 = 10 + G \quad \Rightarrow \quad A = 9 + G$. So $G = 0$ and $A = 9$.
Then we have $B + E = B \Rightarrow E = 0$ (not work because $G = 0$ already) or $1 + B + E = B$
$\Rightarrow 1 + E = 0$ (not true because E is a digit and positive).

Case 2: $1 + A + 1 = 10 + G \quad \Rightarrow \quad A = 8 + G$.

So if $G = 1$, $A = 9$; and if $G = 0$, $A = 8$.

For the subcase $G = 1$, $A = 9$, we have $B + E = 10 + B \Rightarrow E = 10$ (not work because E is a digit less than 10) or $1 + B + E = 10 + B \quad \Rightarrow E = 9$ (not true because A is 9 already).
So we conclude that $G = 0$, $A = 8$.

$$\begin{array}{r} 8BC \\ +1EF \\ \hline 10B1 \end{array}$$

So we have
Case 1: $B + E = 10 + B \Rightarrow \quad E = 10$ (no true since E is a digit less than 10).
Case 2: $1 + B + E = 10 + B \quad \Rightarrow \quad E = 9$.
Since DEF is a prim number, F can only be 3, 7, or 9 while C is 8 (not true because A is 8), 4, or 2, respectively.
So the desired number can be 197 or 199 and both are prime numbers.
So the answer is $197 + 199 = 396$.

30. **Solution:** 6.
Let the numbers be a, b, c with $a \le b \le c$.
We have $a+b+c=12$, and $a^2+b^2+c^2=54$. Or

$a+b=12-c \quad \Rightarrow \quad (a+b)^2=(12-c)^2$ (1)

$a^2+b^2=54-c^2$ (2)

Method 1:

We know that $(a-b)^2 \geq 0$ or $a^2+b^2 \geq 2ab$ (3)

We re-write (1) as $a^2+b^2+2ab=(12-c)^2$ (4)

Substituting (2) into (4): $54-c^2+2ab=(12-c)^2$

$\Rightarrow 2ab=(12-c)^2-(54-c^2)$ (5)

Substituting (2) and (5) into (3): $54-c^2 \geq (12-c)^2-(54-c^2)$

$\Rightarrow 2(54-c^2) \geq (12-c)^2$.

$\Rightarrow 3c^2-24c+36 \leq 0 \quad \Rightarrow \quad c^2-8c+12 \leq 0 \Rightarrow \quad (c-6)(c-2) \leq 0$.

The solution is $2 \leq c \leq 6$.

So the largest possible value for c is 6 when $a = b = 3$.

Method 2:

By Cauchy Inequality,

$a^2+b^2 \geq \frac{(a+b)^2}{2}$ (6)

Substituting (1) and (2) into (6):

$54-c^2 \geq \frac{(12-c)^2}{2} \quad \Rightarrow \quad 3c^2-24c+36 \leq 0 \quad \Rightarrow \quad c^2-8c+12 \leq 0 \quad \Rightarrow$

$(c-6)(c-2) \leq 0$.

The solution is $\Rightarrow \quad 2 \leq c \leq 6$.

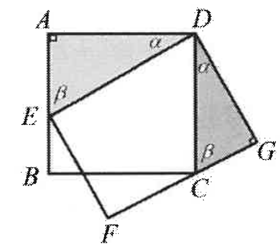
So the largest possible value for c is 6 when $a = b = 3$.

TARGET ROUND SOLUTIONS

1. **Solution:** 64.

Method 1:

Since $\angle ADE + \angle EDC = 90°$, and $\angle GDC + \angle EDC = 90°$, $\angle ADE = \angle GDC = \alpha°$. So $\angle AED = \angle GCD = \beta$.



Therefore triangle ADE is similar to triangle GDC.

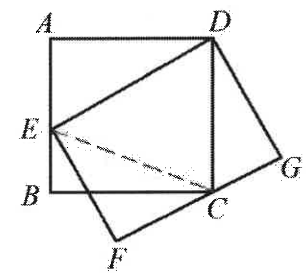
$$\frac{DE}{DC} = \frac{AD}{DG} \Rightarrow \frac{10}{8} = \frac{8}{DG} \Rightarrow \frac{5}{4} = \frac{8}{DG}$$

$$DG = \frac{32}{5}.$$

$$S_{ABCD} = DG \times ED = \frac{32}{5} \times 10 = 64.$$

Method 2:

Connect CE.



$$S_{\Delta ECD} = \frac{1}{2} S_{ABCD}$$

$$S_{\Delta ECD} = \frac{1}{2} S_{DEFG}$$

So $S_{DEFG} = S_{ABCD} = 8 \times 8 = 64$.

2. **Solution:** 21 km.

The truck needs to make 7 trips.

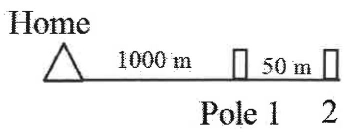
Method 1:

The distances travelled form an arithmetic sequence with first term 1000, common difference 50, and 20 terms in the sequence.

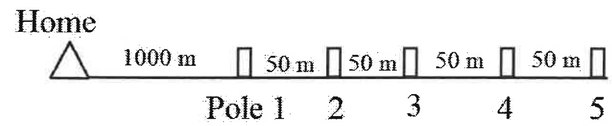
$$S = 2[a_1 + \frac{n(n-1)d}{2}] = 2[1000 + \frac{20(20-1)\times 50}{2}] = 2 \times 10500 = 21000 \text{ meters} = 21 \text{ km}.$$

Method 2:

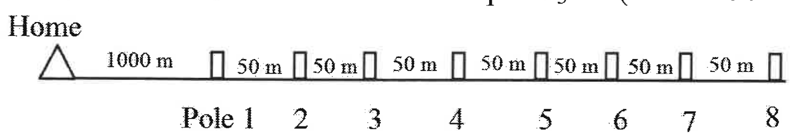
The distance travelled for the first trip is $a_1 = 2(1000 + 50)$



The distance travelled for the second trip is $a_2 = 2(1000 + 50 \times 4)$



The distance travelled for the third trip is $a_3 = 2(1000 + 50 \times 7)$



Similarly, the distance travelled for the last trip is $a_7 = 2(1000 + 50 \times 19)$.
The distances travelled form an arithmetic sequence with first term $a_1 = 2(1000 + 50)$ and the last term $a_7 = 2(1000 + 50 \times 19)$, and 7 terms in the sequence.

$$S = \frac{(a_1 + a_7) \times 7}{2} = \frac{2(1000 + 50) + 2(1000 + 50 \times 19)}{2} \times 7 = 21000 \text{ meters} = 21 \text{ km}.$$

3. **Solution:** 43.
Let the dimension of the cube be $m = n = r = a$.
The number of cubes with two faces painted is $4(m - 2) + 4(n - 2) + 4(r - 2)$.
So we have $4(m - 2) + 4(n - 2) + 4(r - 2) = 492 \quad \Rightarrow \quad 12(a - 2) = 492$
$\Rightarrow \quad a - 2 = 41 \quad \Rightarrow \quad a = 43$.

4. **Solution:** 4,788.

2	x	y	z	16
3	81	x	y	z
	27	18	12	8

$x^2 = 81 \times y$ (1)

$y^2 = xz$ (2)

$z^2 = 16 \times y$ (3)

We also see that $xz = 81 \times 16$ (4)

Substituting (4) into (2): $y^2 = xz = 81 \times 16 \Rightarrow \quad y = 36$.

(1) + (2) + (3): $x^2 + y^2 + z^2 = 81 \times y + xz + 16y$

$= 81\times36+81\times16+16\times36 = 4{,}788$.

5. **Solution:** 756.
We list a few terms to find a pattern:

Terms	1	1	2	3	5	8	13	21
Remainder	1	1	2	0	2	2	1	0

Terms	34	55	89	144	233	377	610	987
Remainder	1	1	2	0	2	2	1	0

Note that the next remainder is also found by adding up the two remainders before it.
So the pattern repeats every 8 terms. In each period we get 3 terms with a remainder 2.

$2016 = 252 \times 8$. So the answer is $252 \times 3 = 756$.

6. **Solution:** 214.
We solve this problem indirectly by finding the natural numbers that do not contain the digit 4 at all.
We have the following digits available: 0, 1, 2, 3, 5, 6, 7, 8, and 9.
We have 8 one-digit numbers.
For two-digit numbers, the first digit can be 1, 2, 3, 5, 6, 7, 8, or 9. The last digit can be any of the 9 digits. So we have $8 \times 9 = 72$ two-digit numbers.

For three-digit numbers, the first digit can only be 1, 2, 3, 5, and 6. The last two digits can be any of the 9 digits.
So we have $5 \times 9 \times 9 = 405$ such three-digit numbers.

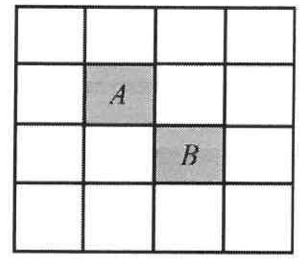
We have one more: 700.

So total we have $8 + 72 + 405 + 1 = 486$ such numbers that do not have the digit 4 at all.
The answer is $700 - 486 = 214$.

7. **Solution:** 44.
Method 1:
1×1 squares: 14.
1×2 squares: $3 \times 4 + 4 = 16$.

1×3 squares: $2 \times 4 = 8$.
1×4 squares: 4.
2×2 squares: 2.
The answer is $14 + 16 + 8 + 4 + 2 = 44$.

Method 2:
The number of rectangles containing the shared area A:
There are just three ways to pick the lower boundary and two ways to pick the top boundary. There are 3 ways to pick the right boundary and 2 ways to pick the left boundary. Their product is



$$\binom{3}{1}\times\binom{2}{1}\times\binom{3}{1}\times\binom{2}{1}=36$$

The number of rectangles containing the shared area B:
$$\binom{2}{1}\times\binom{3}{1}\times\binom{2}{1}\times\binom{3}{1}=36$$

The number of rectangles containing the shared areas A and B:
$$\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}=16$$
The number of rectangles containing the shared areas A or B:
$36 + 36 - 16 = 56$.
The number of rectangles in the figure:
$$\binom{5}{2}\times\binom{5}{2}=100 .$$
The answer is $100 - 56 = 44$.

8. **Solution:** $\frac{3}{2}$.

The total area of these 2016 squares is A and
$$A=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\frac{1}{2015^2}+\frac{1}{2016^2}<(1+\frac{1}{2})^2 .$$
If we use a square with the side length $1+\frac{1}{2}$, it will fit. So the answer is $\frac{3}{2}$.