


23. Set each distinct factor of the equation equal to 0:
 $x^2 - 1 = 0, x^2 - 2 = 0, x^2 - 3 = 0, \dots, x^2 - 20 = 0$.
 The roots are $\pm 1, \pm \sqrt{2}, \dots, \pm \sqrt{20}$. The integral roots are $\pm 1, \pm 2, \pm 3, \pm 4$. There are 8 integers in all.
 A) 4 B) 8 C) 20 D) 40
24. If $2011^{x^2+10x+21} = 1, x^2 + 10x + 21 = (x+7)(x+3) = 0$.
 Thus, $x = -7$ or -3 . The product of -7 and -3 is 21.
 A) -21 B) -10 C) 10 D) 21
25. Since $99n = 3^2 \times 11n$ is a cube, the least such cube is $3^3 \times 11^3$;
 so $n = 3 \times 11^2 = 363$. Finally, $3 + 6 + 3 = 12$.
 A) 27 B) 18 C) 12 D) 9
26. The area is 480, so $w(w + 14) = 480$. Equivalently, $w^2 + 14w - 480 = (w + 30)(w - 16) = 0$. Finally, $w = 16$, and the perimeter is $2(16+30) = 92$.
 A) 88 B) 92 C) 116 D) 172
27. Divide $18x + 27y - 36 = 0$ by 9, then multiply by 2 to get $4x + 6y - 8 = 0$.
 A) 0 B) 12 C) 24 D) 36
28. The graph of $y = |2x - 9| - |2x + 9|$ consists of 3 lines. If $x \geq 9/2$, the graph is the line $y = -18$. If $x \leq -9/2$, the graph is the line $y = 18$. If $-9/2 < x < 9/2$, the graph is the line $y = -4x$. Since y ranges from -18 to 18 , y can equal 15, but y cannot equal any of the other values listed below.
 A) 15 B) 19 C) 20 D) 22
29. The ones digits of powers of 123 are 3, 9, 7, 1, 3,
 Every 4th power ends in 1; so r ends in 1, as does s .
 A) 1 B) 3 C) 7 D) 9
30. The least common multiple of all the integers from 1 through 30 is $2^4 \times 3^3 \times 5^2 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 29$.
 Divide this by $2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 29$, the product of the primes < 30 , to get $2^3 \times 3^2 \times 5 = 360$.
 A) 1 B) 2 C) 12 D) 360



The end of the contest  **A**



Information & Solutions

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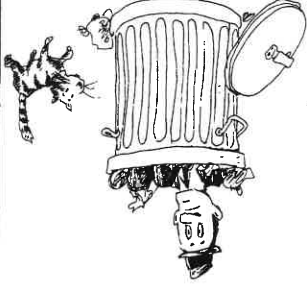
Contest Information

A

- **Solutions** Turn the page for detailed contest solutions (written in the question boxes) and letter answers (written in the *Answer Column* to the right of each question).
- **Scores** Please remember that *this is a contest, and not a test*—there is no “passing” or “failing” score. Few students score as high as 24 points (80% correct); students with half that, 12 points, *deserve commendation!*
- **Answers and Rating Scales** Turn to page 152 for the letter answers to each question and the rating scale for this contest.



12.	C	12. If $n^2 + 5n = 24$ and $n^2 - 4n = -3$, then subtract to get $9n = 24 - (-3) = 27$.
13.	A	13. If $a < 0 < b$, then $1/a$ is negative, $1/b$ is positive, and choice A is false.
14.	C	14. Since $(x^3 - 4x^2 + 4x - 3) \div (x - 3) = x^2 - x + 1$, it is divisible by $x - 3$. Of the polynomials listed, only $x^3 - 4x^2 + 4x - 3$ is divisible by $x - 3$.
15.	A	15. If $(x + 2010)(x + 1) = 0$, then $x = -2010$ or -1 .
16.	C	16. The slope of $xy = x$ is 1. Choice C has slope -1 , the negative reciprocal.
17.	B	17. $y^2 = (x - 5)^2 = x^2 - 10x + 25$. Therefore, $x^2 - 10x + 20 = y^2 - 5$.
18.	D	18. If $(a + b)^2 = a^2 + 2ab + b^2 = 7^2$ and $a^2 + b^2 = 49$, then $2ab = 0$ and $ab = 0$.
19.	B	19. If $x = 2z - y$, then $x + y = 2z$ and $z = (x + y)/2$.
20.	D	20. Let $5x$ and $3x$ be the initial numbers of apple cores and bottles. With 3 more bottles, $5x/(3x + 3) = 3/2$. Simplifying, $10x = 9x + 9$. Therefore, $x = 9$, and $5x = 45$.
21.	A	21. $2x \div 0.2x = 10 = 1000\%$.
22.	D	22. $ 2x + -3x = 2x + 3x = 5x = 5 x $.



1.	D	1. If $xy = 2011^2$, then $(-x)(-y) = (-1)(x)(-1)(y) = xy = 2011^2$.
2.	C	2. One week is 7 days, so w weeks = $7w$ days.
3.	C	3. Since $x^2 - 4x - 12 = (x + 2)(x - 6)$ and $(x + 2)(x - d) = x^2 - 4x - 12$, then $x - 6 = x - d$ and $d = 6$.
4.	A	4. $\sqrt{4x} + \sqrt{9x} + \sqrt{25x} = 2\sqrt{x} + 3\sqrt{x} + 5\sqrt{x} = 10\sqrt{x}$.
5.	B	5. $(x - 2x) + (3x - 4x) + (5x - 6x) + (7x - 8x) + (9x - 10x) = -5x$.
6.	C	6. The sum of five consecutive integers is 165. The middle one is the average = $165 \div 5 = 33$. The integers are 31, 32, 33, 34, and 35.
7.	B	7. $\frac{x}{z} \div \frac{z}{x} = \frac{x}{z} \times \frac{x}{z} = \frac{x^2}{z^2}$.
8.	A	8. Lois and Clark are above the ground. Therefore, $h > 0$, $h + 2 = 28$, and $h = 26$.
9.	D	9. $(x + 5)^2 - (x - 5)^2 = (x^2 + 10x + 25) - (x^2 - 10x + 25) = 20x$.
10.	D	10. If p is a prime between 1000 and 2000, then p is odd. Thus, $p + 567$ is even and is not a prime.
11.	B	11. If $2m + 3s = 86$ and $3m + 4s = 120$, then multiply 1st equation by 4 and 2nd by 3 to get $8m + 12s = 344$ (1) and $9m + 12s = 360$ (2). Subtracting (1) from (2), $m = 16$.

