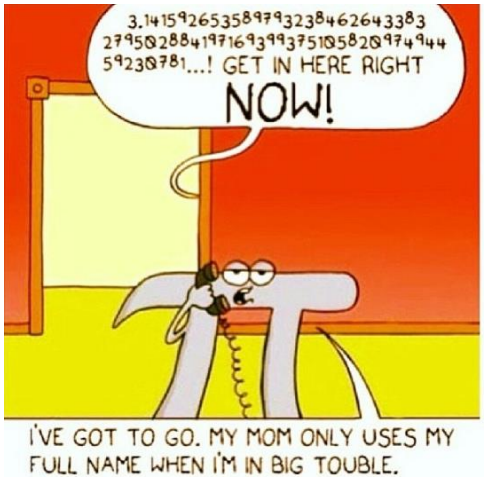
## Integers

**1)** Which two integers have a sum of 15 and a product of 36?

## Largest Product

**2)** What is the largest product that one can obtain by multiplying a two digit number by a three digit number using each of the digits 5, 6, 7, 8, and 9 once the generate the two numbers?



## Hare and Hound

**3)** A hare is 150 paces ahead of a hound that is pursuing him. If the hound covers 10 paces each time the hare covers 6, how many paces will it take for the hound to overtake the hare?

## Lumberjacks

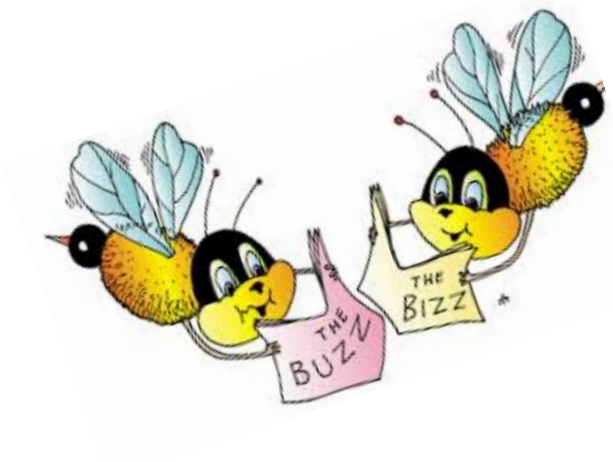
**4)** Two lumberjacks can cut a log into 5 pieces in 30 minutes. At the same average cutting rate, how long will it take the two lumberjacks to cut a similar log into 12 pieces?

## Snacks

**5)** Deterrian looks in the refrigerator for a snack. He finds a bag of baby carrots (4 calories per carrot) and a bag of Snickers Candy Bar Bites (170 calories per Bite). If every carrot is 0.035 pound, how many pounds of carrots would have the same number of calories as one Snickers Candy Bar Bite?


## Bizz Buzz

**6)** To play the math game Bizz Buzz, you begin counting at 1 but skip all multiples of 3 and 5 and all numbers that contain the digits 3 or 5. In this game, the number 16 is the 8th number. If you continue this game, what is the 25th number?



## Input and Output

**7)** In each pair of numbers below, the input is the first number and the output is the second number. After examining the five input and output pairs below, determine the output if the input is -11.

(-1, 3)
(0, 6)
(2, 12)
(10, 36)
(97, 297)

## Steak

**8)** Bob and Maria each purchase steak from the butcher. Maria selects a cut that costs $19.98 per pound. Bob's selection costs $13.98 per pound. If Maria's steak is all meat, and Bob's cut of beef is 1/3 bone and fat, who is paying more for actual steak? (explain your reasoning)

## Prime digits

**9)** What is the greatest two-digit prime number whose two digits are prime and that also sum to a prime number?

## Consecutive odd numbers

**X)** What is the largest number that will always be a factor of the sum of any four consecutive positive odd numbers.

**consecutive** [ k*uh*n-**sek**-y*uh*-tiv ] SHOW IPA

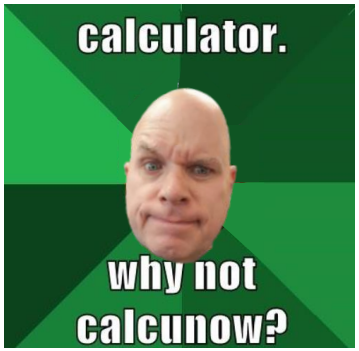
SYNONYMS | EXAMPLES | WORD ORIGIN

*adjective*

1. following one another in uninterrupted succession or order; successive:
   *six consecutive numbers, such as 5, 6, 7, 8, 9, 10.*
2. marked by logical sequence.
3. *Grammar.* expressing consequence or result:
   *a consecutive clause.*

## Calculator confusion

**E)** Alan was asked to multiply 5 by 2, add 2 to the product, divide the sum by 4, and add 8 to the quotient. Alan used his calculator to perform all the calculations in one step. His incorrect answer was 18.5. What was the right answer. What did Alan do wrong?



## Rechargeable batteries

**10)** Instead of buying packages of 20 batteries for $7.00 to power his video game controllers, Nathaniel decides to buy 4 rechargeable batteries and a charger for $15.39. How many times does he need to use the rechargeable batteries to make them a better value?


## Three digit prime

**11)** What is the smallest three-digit prime number whose digits are also prime?

## Middle School Mathematics

**12)** If the letters in the following sequence are repeated indefinitely, what is the 2000$^{th}$ letter in this pattern?

MIDDLESCHOOLMATHEMATICSMIDDLESC…

## Bag of candy

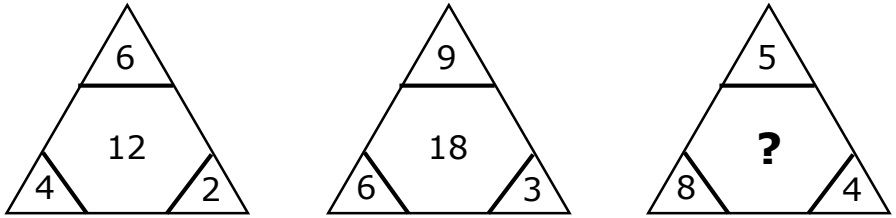
**13)** Mr. Zucker has a bag of chocolate candy containing less than 100 pieces, which he will distribute to the class. If he makes groups with 2, 3, or 4 pieces of candy, he will have 1 piece leftover in the bag. If he makes groups with 5 pieces of candy, he will have no candy left in the bag. How many pieces of candy could be in Mr. Zucker's bag? List all possibilities.

## Crab years

**14)** Suppose that 2 human years are equivalent to 12 crab years. If Mr. Crab spends 2 ½ minutes brushing his teeth, how many human seconds will have elapsed?

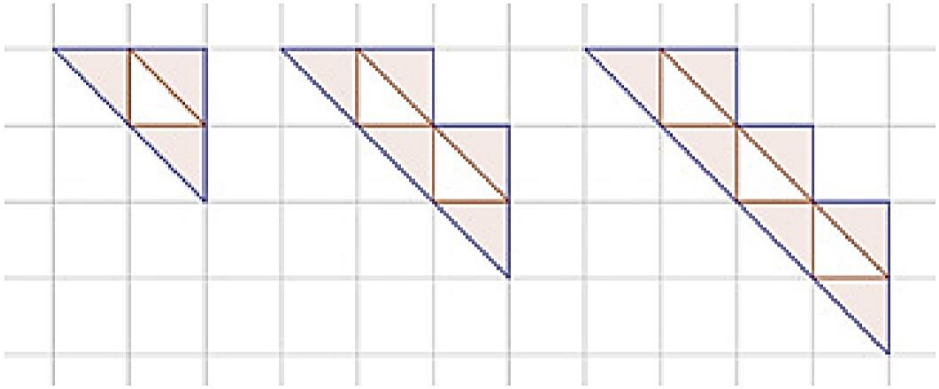
## Patterns and relationships

**15)** What number should replace the question mark?



## Perimeter Pattern

**18)** The unit square grid below shows the first three figures in a growing triangle pattern. If the pattern continues growing in the same way, what will be the outer perimeter (shown in blue below) of the 10th figure? What will be the outer perimeter of the nth figure?



## Frosting cupcakes

**19)** Adi was making cupcakes and found that he could frost 24 cupcakes with 1 1/2 cups of frosting. How many cupcakes could he frost with the full 2-cup container?

## Find the fractions

**1X)** Two positive fractions have a sum of 11/10. Their denominators are 2 and 5. What are the fractions?

## Find the prime

**1E)** Find the greatest three-digit prime number where each of the digits is prime and the sum of all three digits is also prime.

## Follow directions

**20)** Kyle was asked by his math teacher to subtract 4 from a certain number and then divide the result by 10. By mistake, Kyle subtracted 10 from the given number and then divided the result by 4, giving a final answer of 52. What should Kyle's answer have been had he followed the directions correctly?


## Maya and Jase

**21)** Jase and Maya were asked to find the smallest whole number greater than the sum of 5/7, 8/4, and 14/16. While Jase was adding fractions, Maya found the correct answer, which was 4. Jase asked Maya, "What common denominator did you use to add the fractions?" Maya replied, "I didn't need to find a common denominator." How did Maya get the answer?

## Jack and Jill

**22)** Jack and Jill are walking up a hill. Jill walks 6 meters every 4 seconds, and Jack walks 5 meters every 3 seconds. Jill got a head start and walked for 10 seconds and then stopped to wait for Jack. If Jack then starts to walk, how long will it take him to reach Jill?

## John and Jane

**24)** John and Jane are also walking up a hill. John walks 6 meters every 4 seconds, and Jane walks 1 meter every 1 second. Jane got a head start and walked for 10 seconds, and she continued walking once John started. How long will it take John to catch up to Jane, assuming they both maintain a steady rate?

## Find a number

**29)** a) Find the smallest even number other than 2 that is a factor of 78.

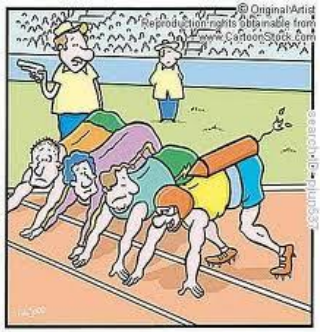
b) Find a prime number < 10 with a square that is > 40 but < 80.

c) Find a 2-digit multiple of 3 whose digits total 6, but neither digit is 1, 2, or 0.

## Who is winning this race?

**2X)** Daryl and Jake are behind Raoul.
Kai is faster than Guy.
There are 2 runners ahead of Hank.
There are 2 runners between Raoul and Jake.
Guy is faster than Jake but slower than Hank.
Hank is behind Kai.
Jake is ahead of Daryl.

Who is winning the race? (Hint: draw a diagram)



## Dice sums

**2E)** Gus is given a standard pair of six-sided dice. How many times must he toss the dice to be certain that he gets the same sum twice?

## 24

**30)** The number 24 is an interesting two-digit number because it is three times the product of its digits, 2 and 4. Find another two-digit number between 0 and 100 that has the same property.


## Always, sometimes, never

**31)** Kanesha says that if you start with any positive fraction and add the same natural number to the numerator and to the denominator, you get a larger fraction. Is Kanesha's assertion always correct, sometimes correct, or never correct?

## More patterns

**32)** Look for patterns to determine the values of the variables *a* and *b*.

| | | |
|---|---|---|
| 7 | 9 | 2 |
| 10 | *a* | 5 |
| 3 | *b* | 3 |

## Change for a dollar

**3E)** Find all the ways you can use exactly 50 coins to make change for a $1.00 bill.

## Cutting corners

**40)** Joanie took a rectangular shaped piece of paper and cut off one corner and disposed of it. What possible shape could the remainder of the rectangle be now?

## My number

**41)** Twice the reciprocal of my number is 7 fewer than the greatest common factor of 90 and 135. What is my number?

## Jack's number

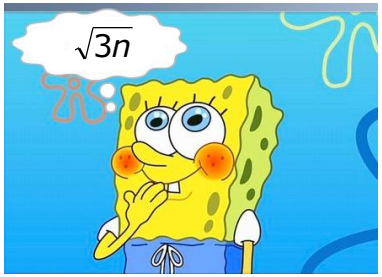
**42)** Jack is thinking of a number. Here are four facts about his number?

a. It is a multiple of 3.
b. It has three digits and is a palindrome (i.e., it has the form ABA).
c. It is odd.
d. At least one digit is 2.

What number(s) satisfy the four properties of Jack's number?

## Square root expression

**4E)** What is the smallest positive integer n so that $\sqrt{3n}$ is a multiple of 4?




## Ages

**50)** The sum of the ages of Travis, Mac, and Ellie is 44. Travis is 4 years older than Ellie, and Mac is twice as old as Ellie. How old is each?

## The Big Friendly Giant


**51)** In Roald Dahl's *The BFG (The Big Friendly Giant),* the butler for the Queen of England is asked to assemble a suitable chair, table and breakfast for the 24-foot giant in the book. The butler reasons that since a 6-foot tall person needs a 2-foot tall chair and a 3-foot table, a 24-foot giant needs an 8-foot chair and 12-foot table.

a. A standard meal for a 6-foot person is 2 eggs, 4 strips of bacon, and 3 pieces of toast. If the same reasoning is used, what meal did the butler calculate for the giant?

b. The giant ate the entire meal in "one titchy little bite." What was wrong with the butler's reasoning in calculating a meal for the giant?

c. What would be an appropriate size meal for the giant?

## M&Ms

**60)** Carly buys a case of M&M's. The company states that there are 54 candies in every pack of M&M's, 48 packs in every box, and two dozen boxes in every case. Carly opens the case and realizes that it is 1 box short and that 1 box in the case is 1 pack short. How many total M&M's were in the case that Carly purchased?

## Consecutive even

**61)** A set of eleven consecutive even whole numbers sums to 374. What is the sum of the first and last even whole numbers in the set?


## Hot chocolate

**70)** Lilia fills a cup with hot-chocolate. She drinks ½ cup and then pours hot water into the cup until it is full. She stirs the new mixture and drinks ½ cup and then fills the cup again with hot water. Lilia continues this pattern until she has consumed 2 full cups. When she next fills the cup with coffee, what percentage of the original hot chocolate remains in her cup?

## Final answer?

**71)** Begin with 8 times the number of days in February in a leap year. Multiply that result by the smallest prime number. From this result, subtract the number of pounds in a ton. Finally, to this last result, add the number of yards in 2 miles. What is your final answer?

## Clearance sale

**72)** Caden found a belt on a clearance rack. The original price was $18. On clearance, everything was 25% off the original price. He also has a coupon for an additional 25% off his purchase. If Caden has $10, does he have enough money to buy the belt?

## Three-digit number

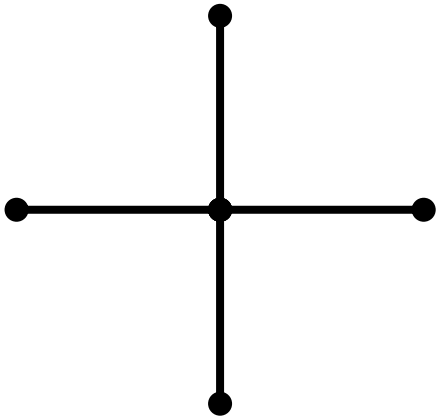
**7E)** I am a three-digit number less than 499. The sum of my digits is a prime number less than 15. The units digit is less than the hundreds digit, and the tens digit is less than the units digit. If none of my digits is a 0, what number, or numbers, can I be?

## Sharing candy

**80)** Cassidy has a bag that contains 120 pieces of candy. Cassidy shares her candy by giving Katelyn 3/4 of 2/3 of her candy. Katelyn shares her candy with Kyle by giving Kyle 3/4 of 2/3 of her candy. If 2/3 of Kyle's candies are red, how many of his pieces are red?

## Make a square

**81)** Four congruent line segments form a "+" sign in the figure. Move one segment to make a square. (Note: This problem involves more than geometry.)



## Going up!

**82)** Lou and Manny use 2 different elevators to reach their offices, each starting on the 1st floor. Lou takes the north elevator to the 25th floor. Manny takes the south elevator to the 5th floor. In terms of floors, how many times farther does Lou travel compared with Manny?

## Five counting numbers

**8E)** A set of five different counting numbers sum to 25. The product of the five numbers in the set is 945. If two of the numbers in the set are 1 and 9, what are the other three numbers?

## Not as easy as it looks

**90)** Each of the letters S, H, E, and I represents a different digit from the list 2, 3, 5, 7. What is the value of each letter? Explain your reasoning.

$$\begin{array}{r} S\,H\,E \\ +\,H\,I \\ \hline S\,I\,S \end{array}$$

## Blinking sloth

**91)** A sloth is a very slow moving animal. Suppose that a blink of an eye for a sloth lasts 3.5 seconds. What is the maximum number of times a sloth could blink in 2 days, 9 hours, and 24 minutes?


"I'm worried that Junior might be hyperactive, he's already blinked three times today."

## Wallpaper

**92)** Ellie is wallpapering her room. The walls are 8 ft. high, and the floor is 10 ft. 5 in. X 11 ft. 2 in. The doorway (7 ft. x 2 ft. 7 in.) and the window (3 ft. x 2 1/3 ft.) will not require wallpaper. How much wallpaper will she need to cover the walls, assuming no waste? Express your answer in square feet.

## Double or Half game

**93)** Here are the rules for the double-or-half game: An amount of money is put in a pot; at the beginning of each round, $1 comes out of the pot; then a coin is tossed.

- If it lands heads, the pot is doubled.
- If it lands tails, only one-half of the money is left in the pot.

You put $4 in the pot and play three rounds. In the first round, heads is tossed; second round, tails; third round, tails. How much money is in the pot?

## Double or half game part 2

**94)** Read the rules above for the double-or-half game. You play three rounds and all three coin tosses are tails. If $3 is in the pot, how much money was in the pot originally?

## Find the number

**99)** Find the number using these clues:

- It is 1 less than a prime number
- It is more than 5 but less than 45.
- The sum of its digits is a prime number.
- It is the largest number that satisfies the three clues above.

## Cookies!

**9E)** Dan baked at least two dozen cookies but fewer than 60 cookies. If he divides the cookies evenly among 7 plates, 4 cookies are left over. If he divides the cookies evenly among 6 plates, 5 cookies are left over. How many cookies did Dan bake?

## Running bases

**X0)** Jake is playing on a regulation-size baseball field. It takes him 4.8 seconds to run the 90 feet from home to first base. Ella runs at the same speed as Jake. How long will it take Ella to run from home to first base on a regulation softball field, which measures 60 feet between bases?

## Expert couponing

**X1)** At an electronics store, earphones cost $20, video games cost $50, and an MP3 player costs $220. You have three coupons to apply to the purchase of these items:

- 15% off any one item
- $10 off any one item
- Any one item is half-price

You can use only one coupon per item. Which coupon should apply to which item so that you minimize the final price? Excluding tax, what is the final price?


## Soccer gear

**X2)** A soccer ball and a jersey (a soccer shirt) together cost $32. If you buy 2 soccer balls and 3 jerseys, the total cost is $86. What does 1 soccer ball cost? What does 1 jersey cost?

## Pattern

**X3)** Determine the next three whole numbers in the following pattern:

2, 4, 6, 12, 14, 28, 30, _____, _____, _____

## Chipmunks

**X4)** As winter approaches, chipmunks begin to gather nuts. The average chipmunk gathers 32 nutshells in 3 days. Unfortunately, 1 out of every 8 nutshells is hollow and does not contain a nut. How many nuts can 3 chipmunks expect to gather in 2 weeks, assuming that 1 out of every 8 nutshells is empty?

## Terms and sequences

**X5)** Consider the following sequence of numbers:

1, 3, 8, 12, 23, 43, 78, …

After the first three terms (1, 3, 8), each subsequent term is found by adding the previous three terms. For example, the fourth term is 1 + 3 + 8 = 12, and the fifth term is 3 + 8 + 12 = 23.

a) Find the next (eighth) term of this sequence.
b) Will the 71$^{st}$ term of this sequence be odd or even? Explain how you know?

## Two rectangles

**X8)** The area of each of two rectangles is 72 square centimeters. The lengths and widths of each rectangle are integers. The length of rectangle 2 is 5 cm. greater than the length of rectangle 1; rectangle 2's width is 15 cm. less than rectangle 1. Find the positive difference in the perimeters of rectangle 1 and rectangle 2.

## Sum problem

**X9)** From the set of the first 20 counting numbers

$$\{1, 2, 3, \ldots, 18, 19, 20\}$$

find the sum of the numbers in the set that are not divisible by 2, 3, or 4.

## Buying gas

**XX)** Daisy has $\frac{1}{8}$ tank of gas remaining when she pulls into a gas station. After she puts 15 gallons of gas in her car, the gas gauge reads $\frac{3}{4}$ full. How many gallons of gas does Daisy's tank hold?

## Midpoint time

**XE)** What time falls exactly between 9:37 p.m. and 2:43 a.m.?

## Train arrival times

**E0)** Suppose that the first of four trains stops in Chicago at 9:37 p.m. and the last train stops at 2:43 a.m. The time between arrivals of consecutive trains is the same. Determine what times the middle two trains arrive.

## Find my number

**E1)** To find my number, take 5 less than the square root of the number found when the product of the first 5 composite numbers is divided by the product of the first 3 prime numbers. What is my number?

## Checkers

**E2)** A traditional checkerboard has 64 squares. Half of the squares are red, and half are black. Each player begins the game with 12 checkers. Your checkers are placed on the 12 black squares on your side of the board, and your opponent's checkers are on the 12 black squares on the other side of the board. Within the first 11 moves, you lose 3 checkers and your opponent loses only 1 checker. After the $11^{th}$ move, what percent of the black squares is occupied by checkers?

## Quarters and dimes

**E3)** Jed comes to school with $4.15 in dimes and quarters in his pocket. If he has more quarters than dimes, what is the smallest number of quarters he can have in his pocket?

## Product of primes

**E4)** What is the smallest positive odd integer you can obtain from the product of 4 different prime numbers if none of the factors is a 7?

## Birds of a feather

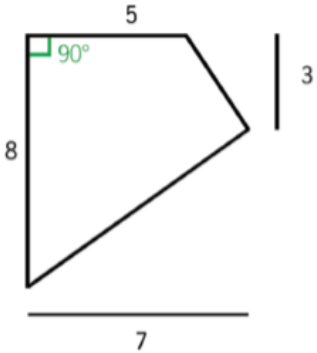
**E5)** At noon, $\frac{1}{4}$ of the birds perched on a fence flew away, but 1 came back. At 1 p.m., $\frac{1}{2}$ of the remaining birds flew away, but 2 came back. Finally, at 2 p.m., $\frac{1}{2}$ of the remaining birds flew off, and 5 birds remained. How many birds were originally perched on the fence before they started leaving at noon?

## Counting number

**E6)** What is the smallest counting number $n$ that would make $12n$ a perfect square?

## Quadrilateral area

**E7)** Find the area of this figure.




## Two-digit multiples of 7

**E8)** There are 13 two-digit multiples of 7. Find the sum of all two-digit multiples of 7 whose digits sum to a prime number.

## Prime factors

**E9)** Find the four unique prime factors of 6555. *Hint*: They are each less than 40.

## Sets

**EX)** Observe the numbers in each of the following sets. Determine the missing number *a* in the last set.

{29, 2, 27}, {28, -3, 33}, {29, 7, 22}, {*a*, 3, 39}

## The chicken and the eggs

**EE)** If four hens lay 9 eggs in 7 days, how many days will it take four dozen hens to lay 27 dozen eggs?

## 100

**100)** Two different sets of two or more consecutive positive integers can be added that sum to 100.

a. Find the set consisting of 5 numbers.

b. Find the set consisting of 8 numbers.

●●

## Relationships

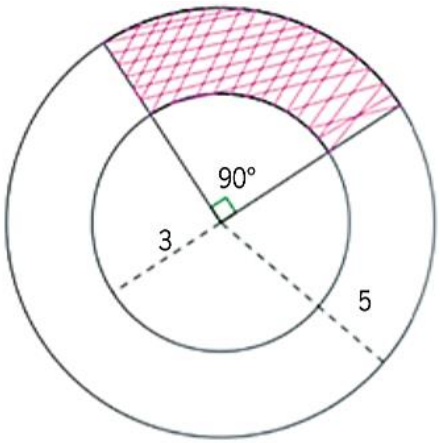
**16)** On the basis of the relationship of the first four pairs of numbers below, what value should replace the question mark in the fifth pair?

(1, 10)
(3, 20)
(5, 30)
(10, 55)
(100, ?)

●●

## Shaded region

**17)** In the figure below, the larger circle has a radius of 5, and the smaller circle has a radius of 3. Find the area of the shaded region.



●●

## Gas Tank

**25)** Wanda realized that she had only 1/8 tank of gas in her car. She drove to a gas station and pumped in 15 gallons, at which point her gas gauge read 3/4 full. How many gallons of gas does Wanda's tank hold?


●●

## Late to Class

**26)** Matt is habitually late to class. His teacher, Mr. Stone, is even later to class 1/4 the time and does not see him arrive late. Of the times he sees Matt arrive late, he forgets to report it 1/3 of the time. Of the times he reports it, 1/2 the time the main office is too busy to record the late arrival. School policy requires a student to have detention after being recorded late three times, so how many times can Matt be late before expecting to have detention?

●●

## Threes and Sevens

**27)** An infinite number of integers can be written as combinations of 3s and 7s.

For example, 10 = 3 + 7, 14 = 2(7), and 38 = 3(8) + 2(7).

What is the largest integer that cannot be written as a combination of 3s and 7s?

●●

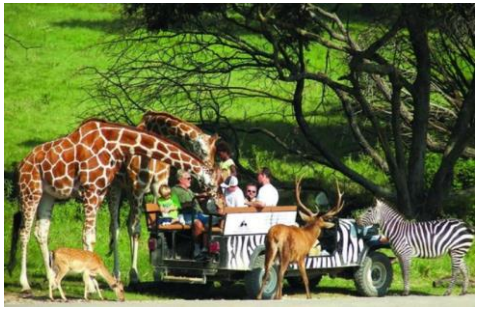
## Green bean farming

**28)** A farmer in Mississippi grows several different crops: 1/2 his acreage is planted in peanuts; he planted 1/4 of the remaining ground in sweet potatoes; after that, he planted okra in 1/2 of what remained. The rest of the garden is planted in green beans. What fraction of his acreage is in green beans?

●●

## Wildlife center

**33)** The Wildlife Rehabilitation Center received a $2000 gift from a private donor to care for injured animals. Another donor pledged $5 for every animal that the center accepts and $8 for every animal that is released into the wild. It costs the center $650 for the government permit to care for wild animals, and $40 a day to care for all the animals. If workers take in and release one animal each day, how long will these monetary gifts support their work?



●●

## Prime Time

**35)** The quotient of the product and sum of three consecutive prime numbers is equal to the largest of the three consecutive prime numbers. What are the three numbers?

●●

## Have it your way

**36)** Students at a school picnic can choose either a hamburger or a hot dog and can choose among ketchup, mustard, and cheese for toppings. If students can have all, some, or none of the toppings, how many different sandwiches can be made?

●●

## Not like the others

**37)** Of the following nine numbers, one is unlike the other eight numbers. Which number does not belong in this 9-number set? Hint: Consider multiplication.

{15, 39, 65, 77, 95, 105, 133, 221, 437}

●●

## Sharing Ice Cream

**38)** Jonah and Olivia share an ice cream treat. When they are halfway through the dessert, Olivia notices that Jonah has been taking 2 spoonfuls for every 1 of her own spoonfuls. Olivia speeds up, so that she is eating 3 spoonfuls to every 1 spoonful that Jonah eats until the treat is gone. Who ate more of the ice cream? (explain your reasoning)

●●

## Alien gathering

**3X)** On Planet Forux there are Forisies and Forlegies. Every Forisie has exactly 4 eyes and exactly 2 legs. Every Forlegie has exactly 4 legs and exactly 1 eye. A gathering of Forisies and Forlegies has an equal number of eyes and legs. What is the minimum number of aliens present?

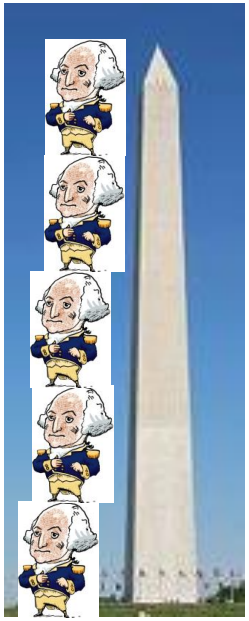
●●

## Forux Day

**43)** On planet Forux one full day lasts 34 hours, 17 hours of light and 17 hours of darkness. It is currently the beginning of the 24th hour (23:00) of the day on both Forux and Earth. After how many hours will the time on Forux and Earth next be the same?

●●

## Washington monument

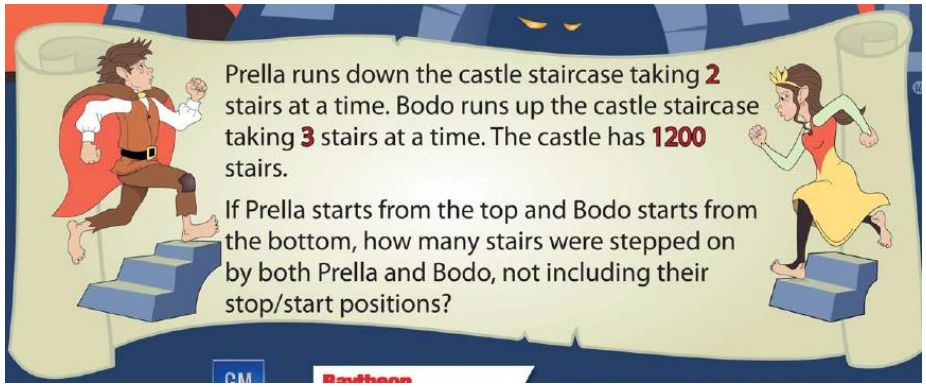


**44)** George Washington was 6’ 2” tall, and the height of the Washington Monument is about 555’ 5”. If you were to stack George Washington one on top of the other (as shown) until the stack first exceeded the top of the Washington Monument, by how many inches would the top-most Washington exceed the height of the monument?

●●

## Prella and Bodo

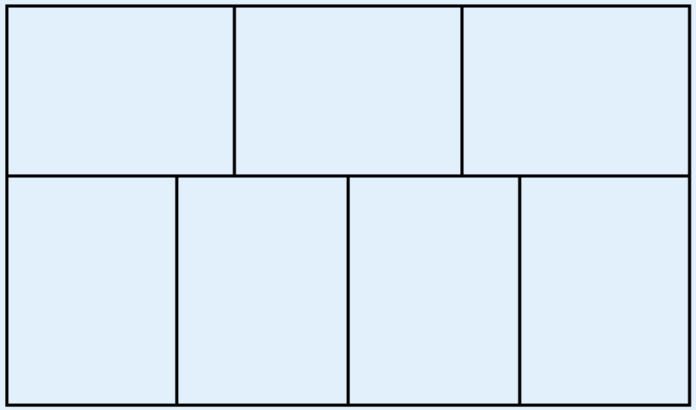
**46)**



●●

## Rectangles

**47)** The large rectangle shown below has an area of 336 square units. It also consists of 7 congruent rectangular regions. Find its perimeter. Explain your reasoning.



●●

## Rectangles, the sequel

**49)** The area of two different rectangles is each 72 square centimeters. The length of the second rectangle is 5 cm greater that the length of the first rectangle and its width is 10 cm less than the width of the first rectangle. Find the dimensions of the two rectangles.

●●

## Aunt Martha's cupcakes

**4X)** Aunt Martha has 5 trays of cupcakes. She describes how they were arranged:

There are 100 total cupcakes on the trays. The first and second trays have 52 cupcakes, the second and third trays have 43 cupcakes, the third and fourth trays have 34 cupcakes, and the fourth and fifth trays have 30 cupcakes.

How many cupcakes are on each tray?

●●

## Unit fractions

**52)** A unit fraction, such as 1/2, 1/11, and 1/25, is a fraction in the form $1/n$, where $n$ is a positive integer. Find a pair of unit fractions such that their difference is 1/72.

●●

## Powers of three

**53)** Observe the following set that contains 26 powers of three:

$$\{3^0, 3^1, 3^2, 3^3, \ldots, 3^{25}\}$$

How many numbers in the set will have a units digit (ones digit) of 1?

●●

## Framed picture

**54)** Mark painted a picture on a 4 in. by 6 in. canvas and decided to frame it. He found a square frame that fits the canvas well and calculated that the shaded area of the frame around the picture (see below) is 1 2/3 times the area of the picture itself. What are the inner dimensions of the frame?

●●

## Lunch combinations

**55)** Each school day, the lunchroom offers you a main course of tacos, pizza, and chicken. Your 4 different drink options are milk, chocolate milk, water, and orange juice. For dessert, you can select chocolate chip cookies or a brownie.

a) How many different lunch combinations can you have if you choose 1 main course, 1 drink, and 1 dessert?

b) If you buy lunch 3 times a week, how many weeks will it take you to try every combination?

●●

## Find a pattern

**58)** Suppose the positive odd numbers are grouped in the following way:

{1}; {3, 5}; {7, 9, 11}; {13, 15, 17, 19}; …

What is the sum of the numbers in the tenth group?

●●

## Train and tunnel

**59)** How long will it take a mile-long train going 20 miles per hour (mph) to drive through and clear a two-mile-long tunnel?

●●

## Candy rewards

**5X)** Mr. Gutbar rewards students with a piece of candy (approximately 60 calories) for every day that they hand in an assignment. Tasha handed in every one of Mr. Gutbar's 140 assignments over the past school year. Given that 30 minutes of play burns 150 calories, how many hours would Tasha have to play to burn the calories from Mr. Gutbar's rewards?

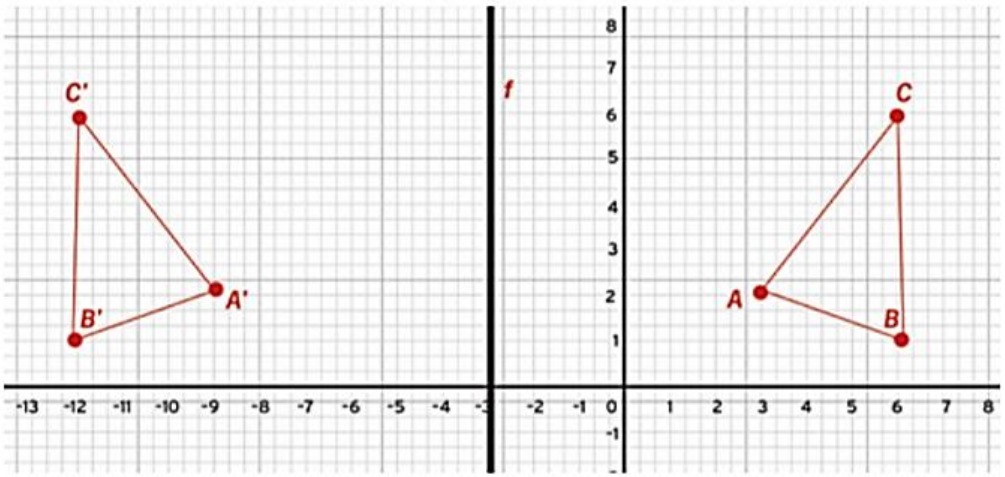
●●

## Haircut

**5E)** Allison goes to the Perpetually Pampered Salon for a 1 p.m. haircut appointment. The salon technician cuts at a rate of 1 hair per second. Assuming that Allison has 98,100 hairs, when will her haircut be finished?

## Reflect on this

**62)** The triangle ABC, pictured below, was reflected over the line f to give triangle A'B'C'. If (x, y) are the coordinates of a point on ABC, what are the coordinates of its image point on A'B'C' ?




## Open Box

**63)** Simon has a collection of 36 square foam pieces that fi t together like puzzle pieces. He wants to use the pieces to build a rectangular box with an open top (i.e., the box has a bottom and 4 sides, but no top). To use all 36 pieces, what dimensions must he make the box?

## What number?

**64)** A number between 1 and 50 meets the following criteria:

it is divisible by 3
the sum of the digits is between 4 and 8
it is an odd number
the product of the digits is between 4 and 8

What is the number?

## Percent of a percent

**65)** The Am I Done Eating Food Company wanted to track repeat buyers of their products in the United States. For, product A, 10% of U.S. households were first-time buyers; 28% of those households bought the product a second time. For product B, 30% of U.S. households were first-time buyers; 9% of those households bought the product a second time. Which product had a greater number of repeat buyers in the United States?

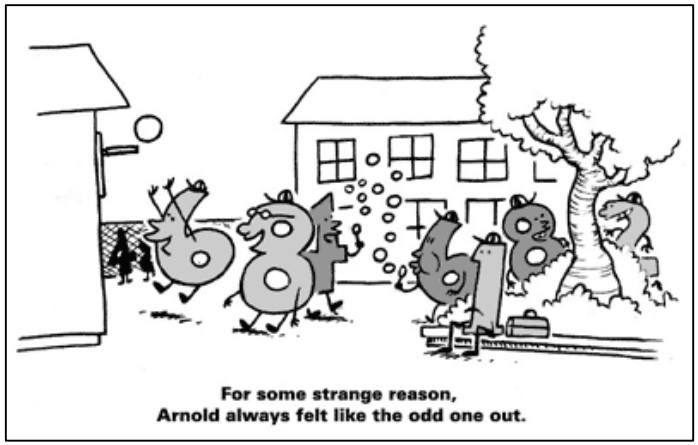
●●

## Odd digits

**68)** How many positive integers in the set:

{50, 51, …, 298, 299}

do not contain any even digits?




●●

## Consecutive integers

**69)** The sum of 11 consecutive positive integers is 2002. What is the greatest of these 11 integers?

## Factorials

**6X)** The expression 4! Is read *four factorial* and is calculated as 4 x 3 x 2 x 1 = 24. The expression 5! Or *five factorial* is 5 x 4 x 3 x 2 x 1 = 120. The number of seconds in 6 weeks is equal to *n*! or *n factorial* for some value of *n*. What is the value of *n*?

## An apple a day

**6E)** On her 23rd birthday, Jasmine decided to live by the adage "An apple a day keeps the doctor away." On her 67th birthday, she calculates the number of apples she has eaten since her 23rd birthday. Since a bushel contains 126 apples, how many bushels of apples has Jasmine eaten?

●●

## At least one two

**73)** How many counting numbers between 100 and 1000 have at least one 2 as a digit?


●●

## Exchanging digits

**74)** A particular three-digit number increases by 9 if we exchange the tens and units digit. The same three-digit number increases by 90 if we exchange the hundreds and tens digits. By how much will the value increase if we exchange the hundreds and units digits?

●●

## 5, 7, 9

**76)** How many different whole numbers less than 1000 can be made using only the digits 5, 7, 9? Digits can be repeated.

●●

## Batting average

**77)** In a double header (2 games), a baseball player got 4 hits out of 7 total times at bat. In the first game, the player got a hit 60% of his times at bat. How many hits did the player have in the second game?

●●

## Which number?

**79)** Which of these numbers is different from all the others? Explain your reasoning.

7158 8102

6104 9145

3129

5147

9156

6137

●●

## Pete the Panther

**7X)** Pete the Panther is the mascot for the Pineville Panthers football team. Each time the Panthers make a touchdown and an extra point (7 points) or kick a field goal (3 points) during a game, Pete does the same number of push-ups as the team's score. For example, if the team scores a touchdown and the extra point and later in the game makes a field goal, Pete will do 7 pushups after the first score and 10 pushups (7 + 3) after the second score. In the last game, the Panthers scored 3 touchdowns (plus the extra point with each touchdown) and 2 field goals, in some order. What would be the greatest number of push-ups that Pete would have to do throughout an entire game? What would be the fewest number of push-ups? Explain how you found your answers.

●●

## 5-digit puzzle

**83)** Find the even 5-digit number that has the following properties:

- None of the digits repeat.
- It is between 20,000 and 77,777, inclusive.
- The digit in the thousands place is triple the digit in the ten-thousands place.
- The units digit is a prime number.
- The hundreds digit is half the sum of the units, thousands, and ten-thousands digit.
- The tens digit is the product of two other digits.

●●

## Candy cost

**85)** A candy store sells only one type of candy, which comes in different colors. Each color is a different flavor, and some flavors cost more than others. Specifically, 1 red piece costs the same as 3 blue pieces, 2 orange pieces cost the same as 5 green pieces, and 2 blue pieces cost the same as 10 green pieces. If 1 orange piece costs 13 cents, how much will 8 red pieces cost?

●●

## Bottles and Boxes

**86)** Hari has several bottles. Of them, 5 are full, 5 are half full, and 5 are empty. He wants to divide these bottles among three boxes that each hold 5 bottles, and he wants each box to weigh the same. How would he divide the bottles? How many different ways can you find?

●●

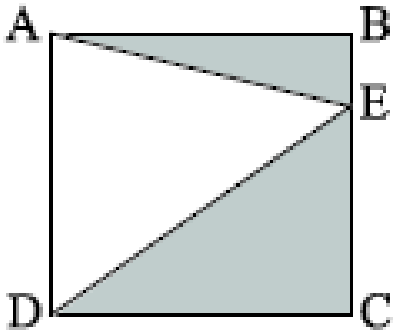
## Locker number

**87)** Inez was assigned a locker at school. She forgot the locker number but remembered that 2 of the 12 positive factors of the locker number are 6 and 25. What is her locker number?


●●

## Find the shaded area

**88)** Square ABCD has sides of length 22 cm. What is the area, in square centimeters, of the region shaded gray?



●●

## Bus route

**8X)** At the first four stops, the number of students that board Mr. Elder's school bus is 4, 6, 12, and 2 students, respectively, and no students exit the bus. After these 4 stops, the number of students on Mr. Elder's school bus is 50% of its full capacity. How many students does Mr. Elder's bus hold at full capacity?

After the last stop on Mr. Elder's route, the number of students on the bus is ¾ of its full capacity. Once Mr. Elder has completed picking up all the students on his route, if no students have exited the bus, how many students could the remaining empty seats accommodate?

There are a total of 8 stops on Mr. Elder's school bus route. How many students are picked up at each of the last four stops if the same number of students boards the bus at each of these four stops, and no students exit the bus?

●●

## Speed limit

**95)** Sarah and James drove from point A to point B along different routes. Sarah's route was 9 miles, and she traveled at an average speed of 45 miles per hour (mph). James' route was 2 miles longer. Sarah and James arrived at the same time.

Sarah claimed that James must have broken the speed limit of 55 mph, but James insisted that he did not.


Who is correct?

●●

## @ operation

**96)** Use these results of @ as an operator:

1 @ 6 = 1
2 @ 4 = 16
3 @ 5 = 243
4 @ 7 = 16,384

Find the value of 5 @ 9.

●●

## Look for a pattern

**97)** What is the units (ones) digit of $3^{722}$ ?

●●

## Computer profit

**98)** If Alex sells his computer for $2392, he makes a profit of 30%. What price should he charge if he wants a profit of 40%?

"Somebody broke into your computer, but it looks like the work of an inexperienced hacker."

●●

## Stack of nickels

**X6)** A stack of 25 U.S. nickels has a height of approximately 1.925 inches. What is the value in dollars of a 7-foot-high stack of U.S. nickels? Express your answer to the nearest $0.05.

●●

## Find all numbers…

**X7)** Find all numbers that satisfy *all* the following conditions:

a. Use positive whole numbers less than 100.
b. Four more than each number is a multiple of 6.
c. The sum of the digits of each number is a multiple of 4.
d. Two-digit numbers where the tens digit is greater than the units (ones) digit.

●●●

# Sailors, Coconuts, and Monkeys

**23)** Three sailors, marooned on an island, spent the day collecting coconuts. The sailors were too tired at night to count them, so they agreed to divide them equally the next morning.

- During the night, one sailor woke up and decided to take his share. He divided the coconuts into three equal piles. One coconut was leftover, which he threw to the monkeys on the island. He took his share and left the remaining coconuts in one pile.

- Later that night, the second sailor awoke and. Likewise, decided to take his share. He, too, made three equal piles and had one coconut leftover, which went to the monkeys. He took his share, and made one pile out of the coconuts.

- Before dawn, the third sailor awoke and did the same thing with the remaining coconuts.

In the morning, all three sailors saw the small pile but said nothing. When they divided the remaining coconuts equally, each sailor received seven coconuts; the one leftover went to the monkeys. How many coconuts were in the original pile?

●●●

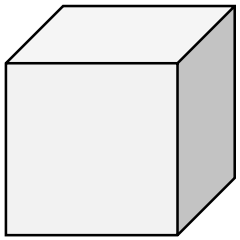
## Blinking

**34)** From birth, a typical person blinks an average of 25 times per minute when awake. When Katelyn was 10 years old, she blinked an average of 27 times per minute. When she was 11, she blinked 26 times per minute. When she was 12, she blinked 25 times per minute. At 10 years old, Katelyn averaged 9 hours, 45 minutes of sleep each day; at age 11, she slept 9 hours, 30 minutes; and at age 12, she slept 9 hours, 15 minutes. What is the sum of the number of times Katelyn blinked when she was 10, 11, and 12 years old? (Calculate using 365 days in a year.)

●●●

## Think outside the cube

**39)** Each of the six identical faces of a cube is to be painted either red or white. How many distinct cubes can be created?



●●●

# Abe’s bag of money

**45)** Abe has a bag containing 340 coins and 160 bills totaling $1,809. There are pennies, nickels, fives, tens, and twenties. The total value of the tens is equal to the total value of the twenties. There are the same number of fives and twenties. What is the total value of the coins and bills bearing Lincoln’s portrait?

●●●

# Pascal's triangle

**48)** Use the pyramid of numbers below, known as Pascal's triangle, to answer these questions:

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

a) What are the first two numbers in row 33?

b) How many odd numbers will be in the 33rd row? (Hint: Add more and more rows until you see a pattern.)

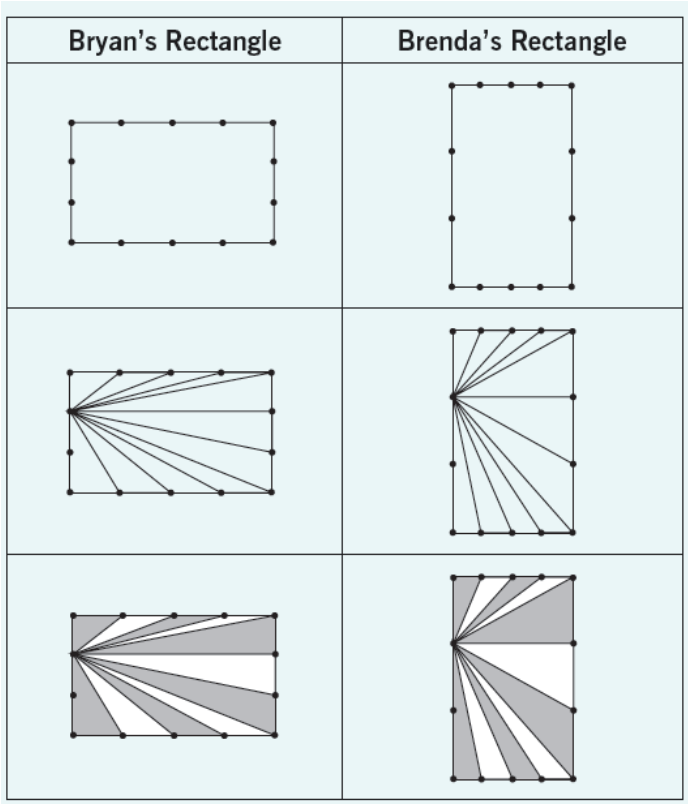
●●●

# Shaded rectangles

**56)** Follow these directions:

- Draw a rectangle
- Divide the sides of the height into 3 congruent line segments
- Divide the sides of the base into 4 congruent line segments;
- Draw a line segment to each point in the rectangle from the point just below the top-left corner
- Shade every other section, starting at the top left and moving clockwise.

What fractions of Bryan's and Brenda's rectangles are shaded?



●●●

# Shaded squares

**57)**

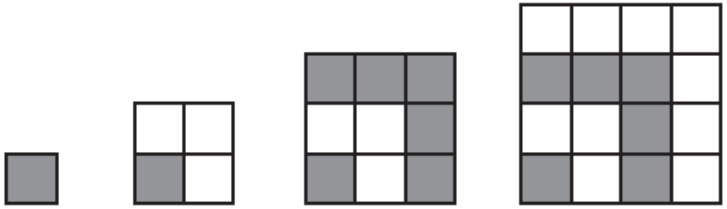


Figure 1 Figure 2 Figure 3 Figure 4

The first four figures of a pattern are shown. The lines in each figure are equally spaced so that each is composed of one or more squares. If the pattern continues, how many shaded squares will be in the 10th figure?

What percent of the 10th figure will be unshaded?

Each of the smaller squares has sides of length 3 units. For example, Figure 2 has a total area of $6 \times 6 = 36$ units$^2$. What is the sum of the areas of the shaded regions in the first 10 figures of the pattern?

●●●

# Earth Day flag

**66)** Did you know that Earth Day even has it's own flag? It has 13 horizontal stripes (6 white and 7 green) and has a green square in the upper left corner that contains the Greek letter theta displayed in yellow.

If an earth day flag is 4 feet by 6 feet and all stripes have equal width, how many square inches of white are on the front of the flag? As you make your calculations, it is OK to round your decimals to the nearest tenth.

●●●

## Bookstore coupons

**67)** Six friends (Anita, Bill, Catherine, Diana, Evan, and Fred) go to a bookstore. Each friend is given a coupon for a certain percentage off the price of one book.

(a) Anita notices that the sum of the percents off listed on their six coupons is 100.
(b) Bill notices that his coupon and Anita's coupon list the same percent off.
(c) Catherine points out that her percent off is the same as Diana's and that Bill's percent off is three times the size of Diana's.
(d) Evan is happy with his 28% off coupon.
(e) Fred is even happier, since his coupon's percent off is 10 times that of Diana's coupon.

What is the percent off for each of the six friends' coupons?

●●●

## Cat and mouse

**75)** In the video game Cat and Mouse, you receive 4 points for each cat caught and 11 points for each mouse caught.

a) What scores are impossible to get in this game?
b) What pattern do you see in the impossible scores?

●●●

# Puppy Love

78)

## Puppy Love

Ian's beagle had a litter of puppies. Ian, a budding statistician, decided to keep track of some data about the puppies. Here is some information Ian collected about his puppies:

- There were 4 female and 3 male puppies in the litter.
- The median birth weight of the puppies was 14 ounces.
- The mean birth weight of the puppies was 15 ounces.
- The mode birth weight of the puppies was 14 ounces.
- The range of the birth weights was 8 ounces.

1. Determine the possible weights for Ian's puppies. Find another set of puppy weights that matches the information given. Explain your strategy.

2. Ian weighed the puppies 1 week after they are born. He discovered that all the puppies had gained exactly 8 ounces. How will the median, mean, mode, and range of the puppies' weights compare with those of the puppies' weights at birth?

3. Ian weighed the puppies again after another week. He discovered that 3 of the puppies had each gained 10 ounces, 2 puppies had each gained 16 ounces, and 2 puppies that were sick each lost 3 ounces. What was the mean weight of the puppies after this last weighing?

●●●

# The Three Bears Cookie Store

**84)** The Bear family decided to open a cookie store that featured bear-sized cookies. Mama and Baby Bear decorated the cookies, and Papa Bear sold the cookies. Mama Bear could completely decorate a cookie every 7 minutes. Baby Bear could completely decorate a cookie every 12 minutes. Unfortunately, Papa Bear was eating one of the cookies every 9 minutes. If they open the store and have no cookies decorated, how long will it take them to produce 500 decorated cookies that can be sold?

●●●

# Making Squares

89)

## Making Squares

Ms. Harper needs a large number of congruent squares for a craft project for her students. She finds a large piece of cardboard that measures 588 cm × 630 cm. She would like to use all the cardboard, with no waste, and wants the squares to be as large as possible.

1. What are the dimensions of the largest squares that Ms. Harper can create without having any leftover cardboard? Remember that all squares must be congruent. How do you know that no other larger size is possible?

2. How many squares of that size will Ms. Harper be able to create?

   Ms. Harper decides that a nonsquare rectangle might work better for her students' craft project. She cuts the 588 cm × 630 cm piece of cardboard into 108 congruent rectangles with no waste.

3. What are the dimensions of the rectangles that Ms. Harper could create? Are there any other dimensions she could have used? How do you know?

●●●

# Fund Run

**9X)** Aaron, Beth, Candace, David, and Emil ran around a track as part of a fundraiser for their school. Aaron ran first, and then when he was tired, Beth ran. When Beth got tired, Candace ran, and so on. By the end of the fundraiser, they had run a total of 30 miles.

Aaron and Beth combined ran 40 percent of the total distance. Beth and Candace combined ran 34 percent of the total distance. Candace and David combined ran 40 percent of the total distance. And David and Emil combined ran 41 percent of the total distance.

How far did each person run?

**Please let Mr. Colby know that you took the last sheet.**

**Thank you!**