Curricular Conversations

Are You Smarter Than a 4th Grader?

Trinity Episcopal School

February 12, 2014

Try the following problem:

89 x 25

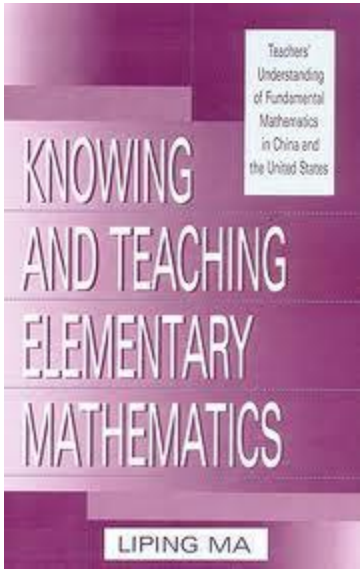
Is this how you solved it?

1
~~4~~
89
X25
445
+1780
2225

How Would a Mathematician Solve It?

Ann Dowker (1992) asked 44 mathematicians to estimate several typical multiplication and division computation problems, one of which was 76x89, and assessed their strategies. Only 4% of the responses represented the use of the standard algorithm. **The mathematicians looked at the numbers first and then found elegant, efficient strategies that seemed appropriate for the numbers.** They made the numbers friendly (often by using landmark numbers), and they played with relationships.

How Do Students in Other Countries Solve It?



Liping Ma (1999) compared the way Chinese and American teachers think about and teach the multiplication algorithm and how they work with students who make place value mistakes. Most Chinese teachers approach the teaching of the multiplication algorithm conceptually. They explain the distributive property and break the problem up into the component problems.

89 x 25

=(80+9) (20+5)

=(80 x 20) + (80 x 5) + (9 x 20) + (9 x 5)

=1,600 + 400 + 180 + 45

=2,225

Traditional Algorithm

In contrast, 70% of American teachers teach the algorithm as a series of procedures and interpret students' errors as a problem with regrouping and lining up.

Constance Kamii (1993), a well-known researcher in the area of mathematics, suggests that "teaching the traditional algorithm can be harmful to students as they have to give up their own meaning-making in order to adopt the teacher's procedures."

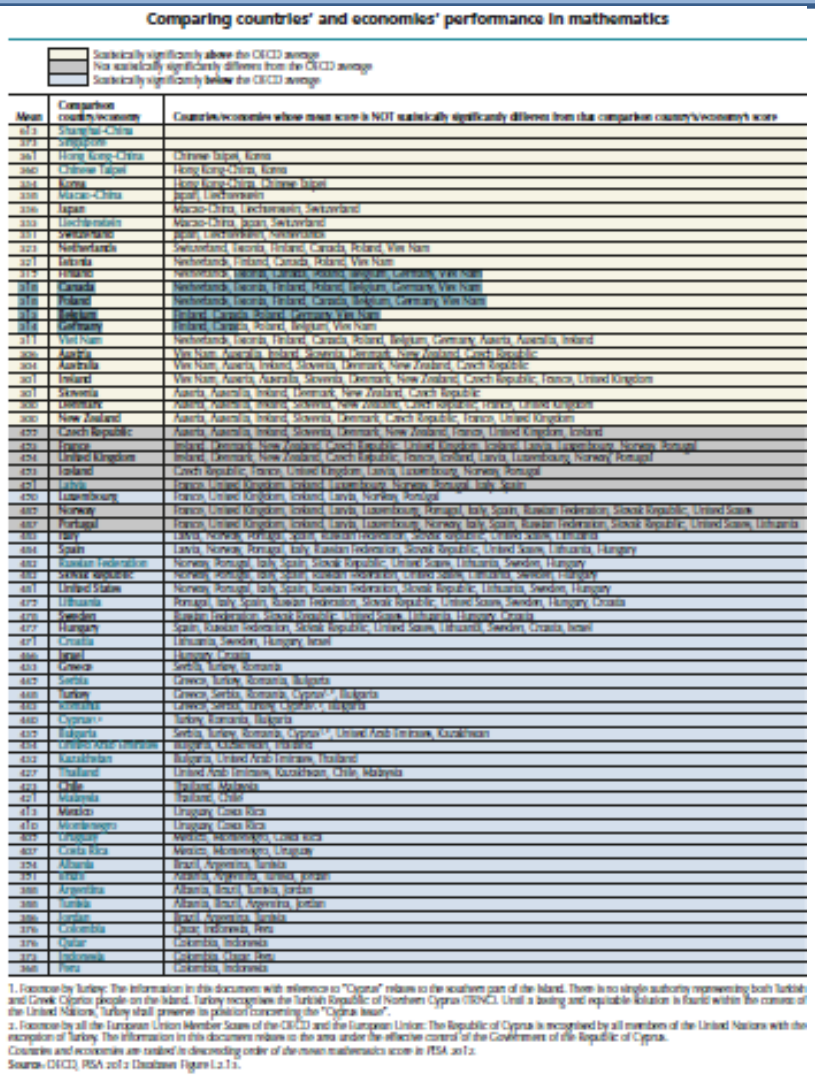
PISA

OECD Average
494

U.S.
481

	Mean score in PISA 2012
OECD average	494
Shanghai-China	613
Singapore	573
Hong Kong-China	561
Chinese Taipei	560
Korea	554
Macao-China	538
Japan	536
Liechtenstein	535
Switzerland	531
Netherlands	523
Estonia	521
Finland	519
Canada	518
Poland	518
Belgium	515
Germany	514
Viet Nam	511
Austria	506
Australia	504
Ireland	501
Slovenia	501
Denmark	500
New Zealand	500
Czech Republic	499
France	495
United Kingdom	494
Iceland	493
Latvia	491
Luxembourg	490
Norway	489
Portugal	487
Italy	485
Spain	484
Russian Federation	482
Slovak Republic	482
United States	481
Lithuania	479
Sweden	478
Hungary	477
Croatia	471
Israel	466
Greece	453
Serbia	449
Turkey	448
Romania	445

PISA



Comparing countries' and economies' performance in mathematics

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

Mean	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from the comparison country's/economy's score
613	Shanghai-China	
573	Singapore	
561	Hong Kong-China	Chinese Taipei, Korea
560	Chinese Taipei	Hong Kong-China, Korea
554	Korea	Hong Kong-China, Chinese Taipei
536	Macao-China	Japan, Liechtenstein
536	Japan	Macao-China, Liechtenstein, Switzerland
535	Liechtenstein	Macao-China, Japan, Switzerland
531	Switzerland	Japan, Liechtenstein, Netherlands
523	Netherlands	Switzerland, Estonia, Finland, Canada, Poland, Viet Nam
521	Estonia	Netherlands, Finland, Canada, Poland, Viet Nam
519	Finland	Netherlands, Estonia, Canada, Poland, Belgium, Germany, Viet Nam
518	Canada	Netherlands, Estonia, Finland, Poland, Belgium, Germany, Viet Nam
518	Poland	Netherlands, Estonia, Finland, Canada, Belgium, Germany, Viet Nam
515	Belgium	Finland, Canada, Poland, Germany, Viet Nam
514	Germany	Finland, Canada, Poland, Belgium, Viet Nam
511	Viet Nam	Netherlands, Estonia, Finland, Canada, Poland, Belgium, Germany, Austria, Australia, Ireland
506	Austria	Viet Nam, Australia, Ireland, Slovenia, Denmark, New Zealand, Czech Republic
504	Australia	Viet Nam, Austria, Ireland, Slovenia, Denmark, New Zealand, Czech Republic
501	Ireland	Viet Nam, Austria, Australia, Slovenia, Denmark, New Zealand, Czech Republic, France, United Kingdom
501	Slovenia	Austria, Australia, Ireland, Denmark, New Zealand, Czech Republic
500	Denmark	Austria, Australia, Ireland, Slovenia, New Zealand, Czech Republic, France, United Kingdom
500	New Zealand	Austria, Australia, Ireland, Slovenia, Denmark, Czech Republic, France, United Kingdom
499	Czech Republic	Austria, Australia, Ireland, Slovenia, Denmark, New Zealand, France, United Kingdom, Iceland
496	France	Ireland, Denmark, New Zealand, Czech Republic, United Kingdom, Iceland, Latvia, Luxembourg, Norway, Portugal
494	United Kingdom	Ireland, Denmark, New Zealand, Czech Republic, France, Iceland, Latvia, Luxembourg, Norway, Portugal
493	Iceland	Czech Republic, France, United Kingdom, Latvia, Luxembourg, Norway, Portugal
491	Latvia	France, United Kingdom, Iceland, Luxembourg, Norway, Portugal, Italy, Spain
490	Luxembourg	France, United Kingdom, Iceland, Latvia, Norway, Portugal
489	Norway	France, United Kingdom, Iceland, Latvia, Luxembourg, Portugal, Italy, Spain, Russian Federation, Slovak Republic, United States
487	Portugal	France, United Kingdom, Iceland, Latvia, Luxembourg, Norway, Italy, Spain, Russian Federation, Slovak Republic, United States, Lithuania
485	Italy	Latvia, Norway, Portugal, Spain, Russian Federation, Slovak Republic, United States, Lithuania
484	Spain	Latvia, Norway, Portugal, Italy, Russian Federation, Slovak Republic, United States, Lithuania, Hungary
482	Russian Federation	Norway, Portugal, Italy, Spain, Slovak Republic, United States, Lithuania, Sweden, Hungary
482	Slovak Republic	Norway, Portugal, Italy, Spain, Russian Federation, United States, Lithuania, Sweden, Hungary
481	United States	Norway, Portugal, Italy, Spain, Russian Federation, Slovak Republic, Lithuania, Sweden, Hungary
477	Lithuania	Portugal, Italy, Spain, Russian Federation, Slovak Republic, United States, Sweden, Hungary, Croatia
478	Sweden	Russian Federation, Slovak Republic, United States, Lithuania, Hungary, Croatia
477	Hungary	Spain, Russian Federation, Slovak Republic, United States, Lithuania, Sweden, Croatia, Israel
471	Croatia	Lithuania, Sweden, Hungary, Israel
466	Israel	Hungary, Croatia
453	Greece	Serbia, Turkey, Romania
447	Serbia	Greece, Turkey, Romania, Bulgaria
448	Turkey	Greece, Serbia, Romania, Cyprus[1,2], Bulgaria
445	Romania	Greece, Serbia, Turkey, Cyprus[1,2], Bulgaria
440	Cyprus[1,2]	Turkey, Romania, Bulgaria
439	Bulgaria	Serbia, Turkey, Romania, Cyprus[1,2], United Arab Emirates, Kazakhstan
434	United Arab Emirates	Bulgaria, Kazakhstan, Thailand
432	Kazakhstan	Bulgaria, United Arab Emirates, Thailand
427	Thailand	United Arab Emirates, Kazakhstan, Chile, Malaysia
423	Chile	Thailand, Malaysia
421	Malaysia	Thailand, Chile
413	Mexico	Uruguay, Costa Rica
410	Montenegro	Uruguay, Costa Rica
409	Uruguay	Mexico, Montenegro, Costa Rica
407	Costa Rica	Mexico, Montenegro, Uruguay
394	Albania	Brazil, Argentina, Tunisia
391	Brazil	Albania, Argentina, Tunisia, Jordan
388	Argentina	Albania, Brazil, Tunisia, Jordan
388	Tunisia	Albania, Brazil, Argentina, Jordan
386	Jordan	Brazil, Argentina, Tunisia
376	Colombia	Qatar, Indonesia, Peru
376	Qatar	Colombia, Indonesia
375	Indonesia	Colombia, Qatar, Peru
368	Peru	Colombia, Indonesia

1. Footnote by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".

2. Footnote by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Countries and economies are ranked in descending order of the mean mathematics score in PISA 2012.

Source: OECD, PISA 2012 Database, Figure I.2.13.

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Multiplication Using Distribution

89 x 25

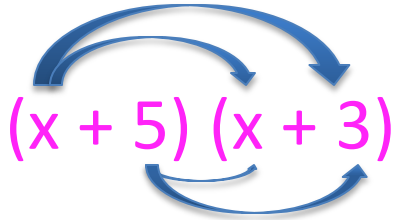
=(80+9) (20+5)

=(80 x 20) + (80 x 5) + (9 x 20) + (9 x 5)

=1,600 + 400 + 180 + 45

=2,225

Having a solid understanding of the distributive property makes the transition to Algebra seamless.

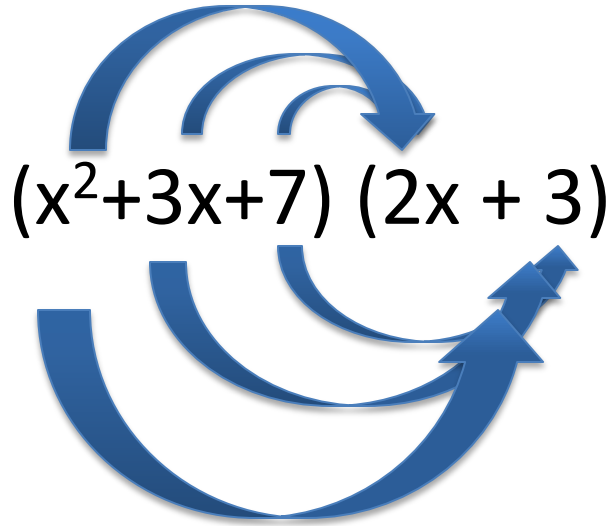


Do you remember the FOIL method in Algebra?

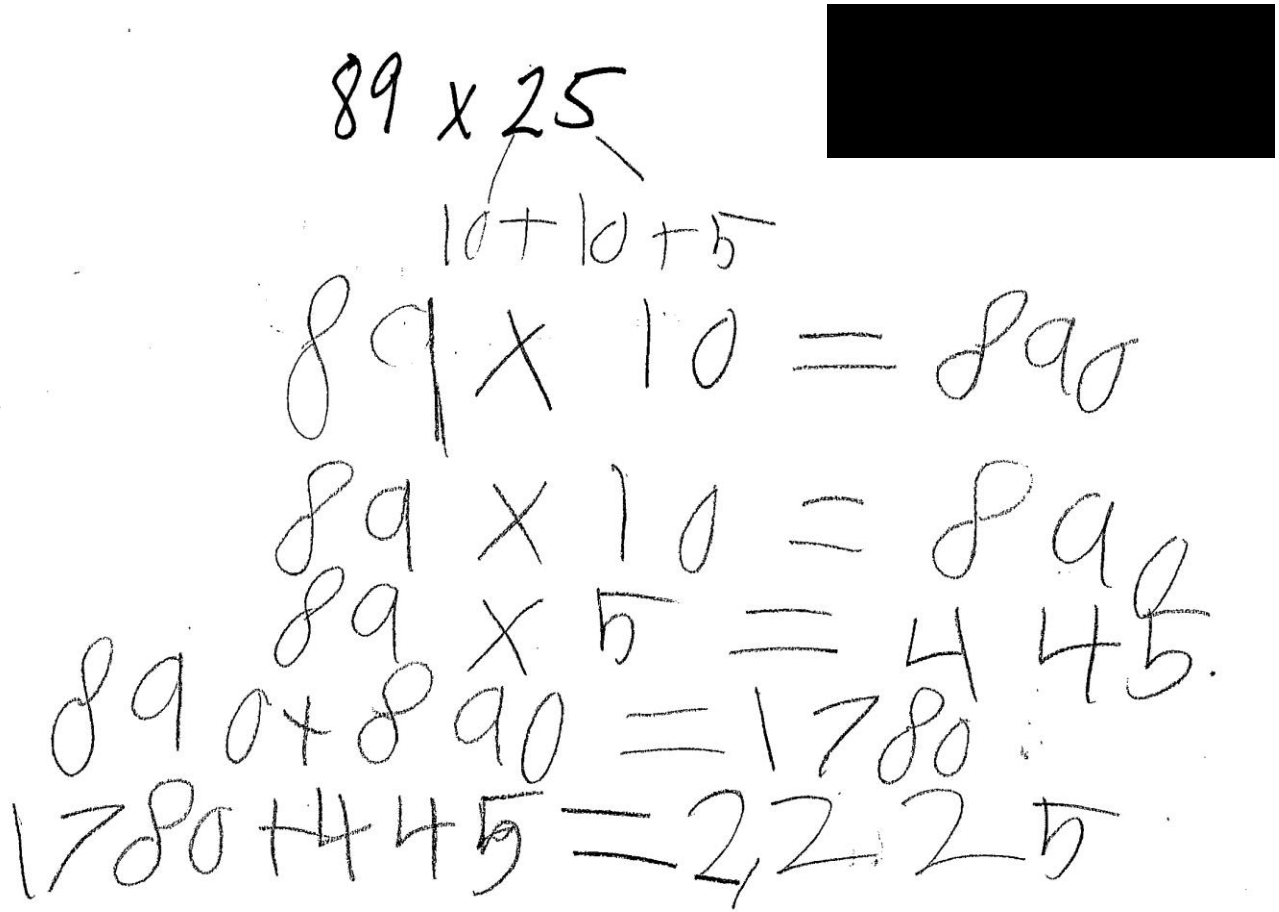
(x•x) + (x•3) + (x•5) + (5 •3)

Distribution

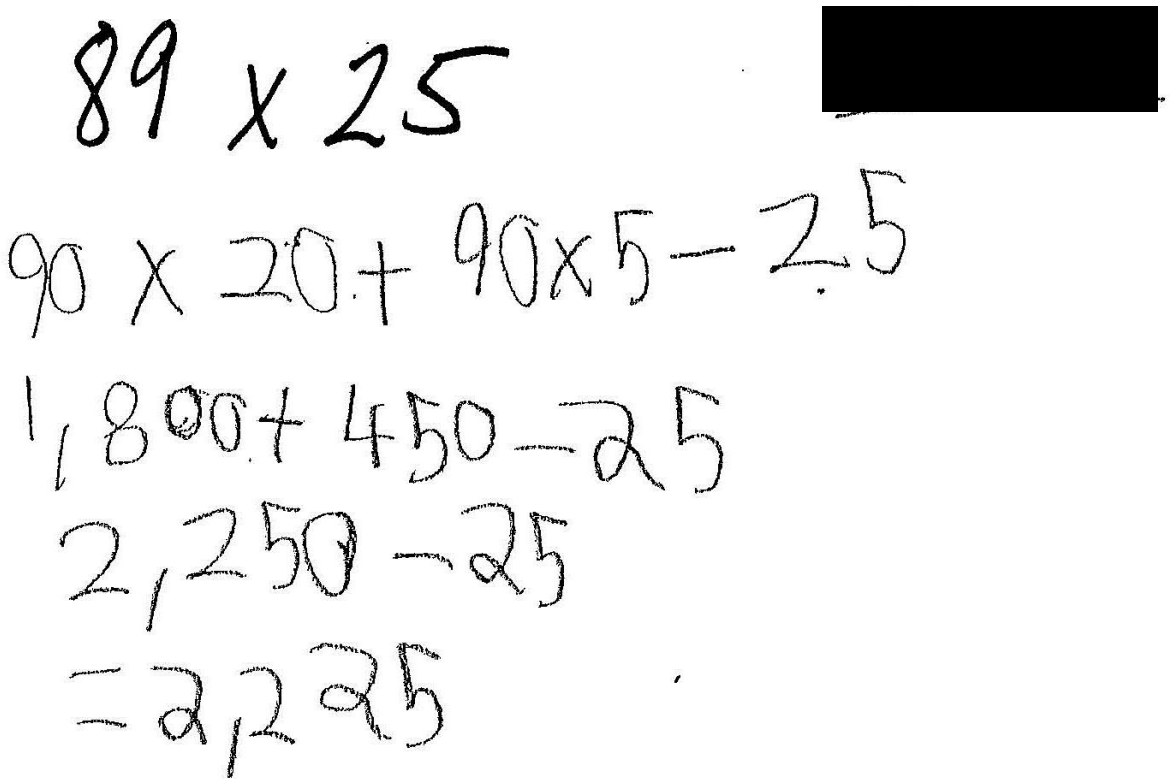
Knowing FOIL doesn't help with this expression.
But knowing distribution does!



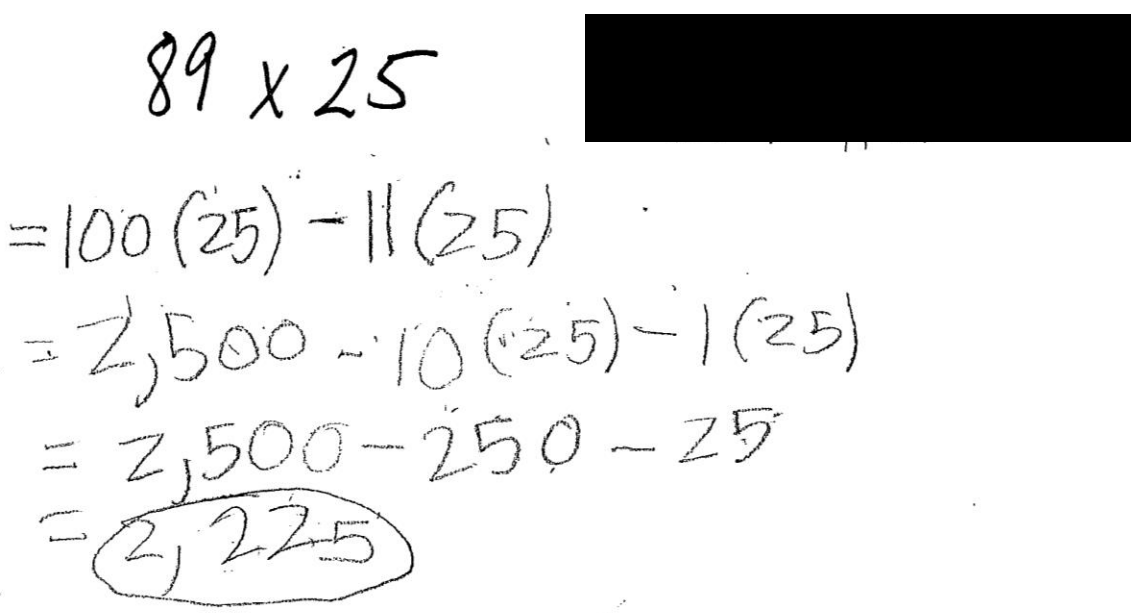
Student - Example A



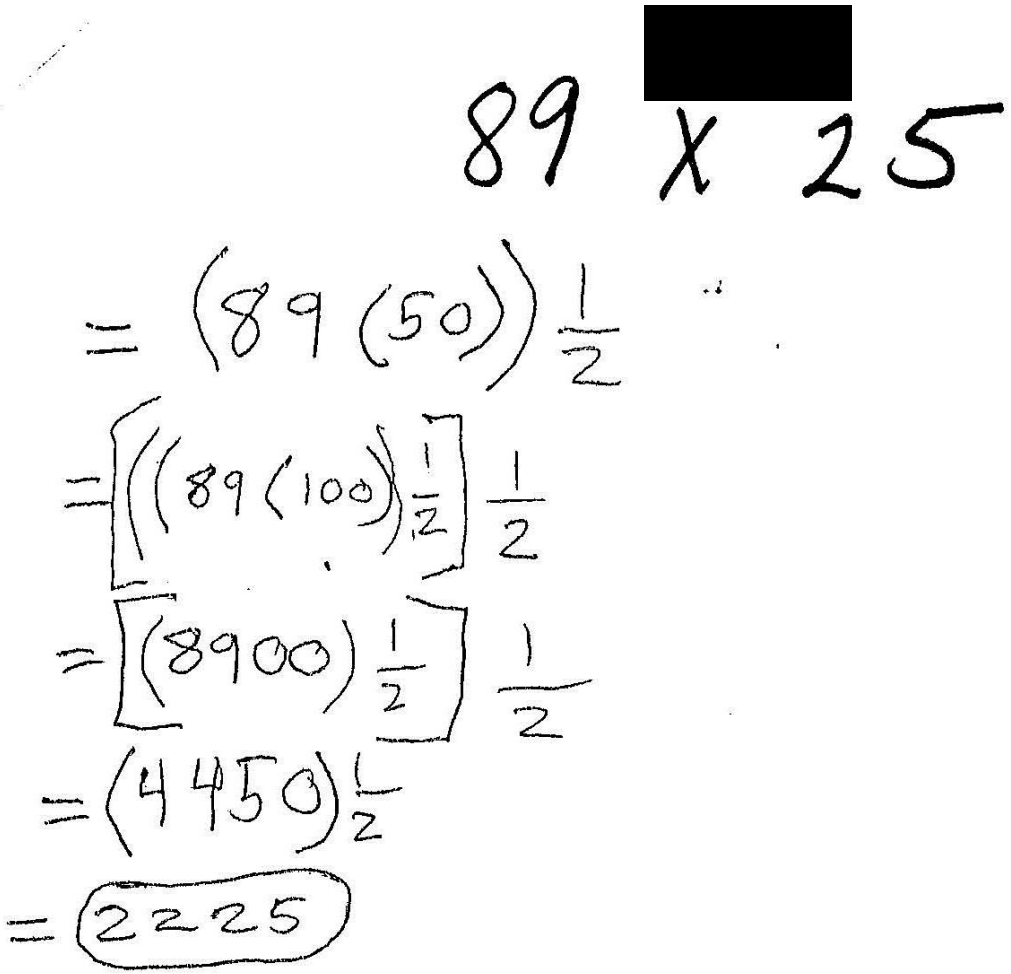
Student - Example B



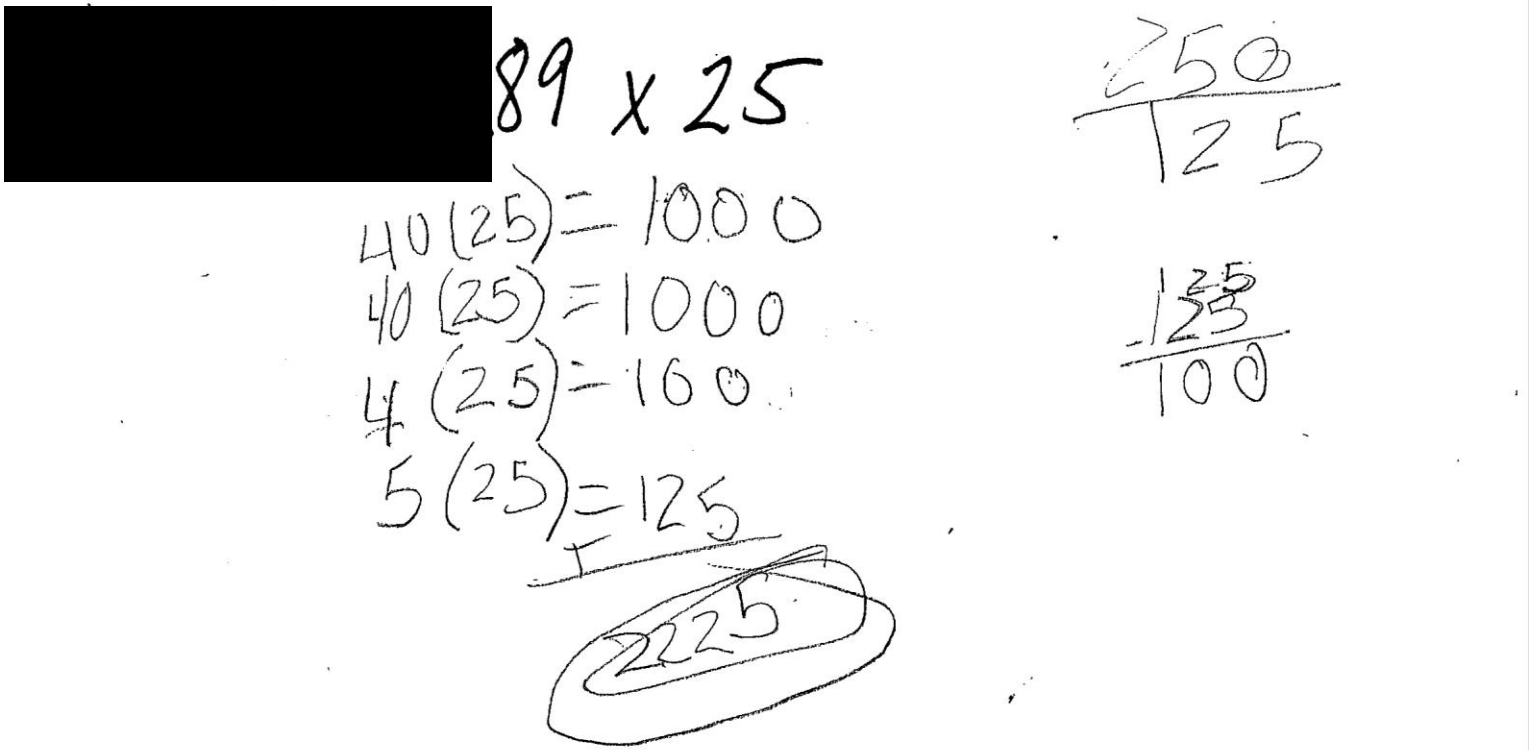
Student – Example C



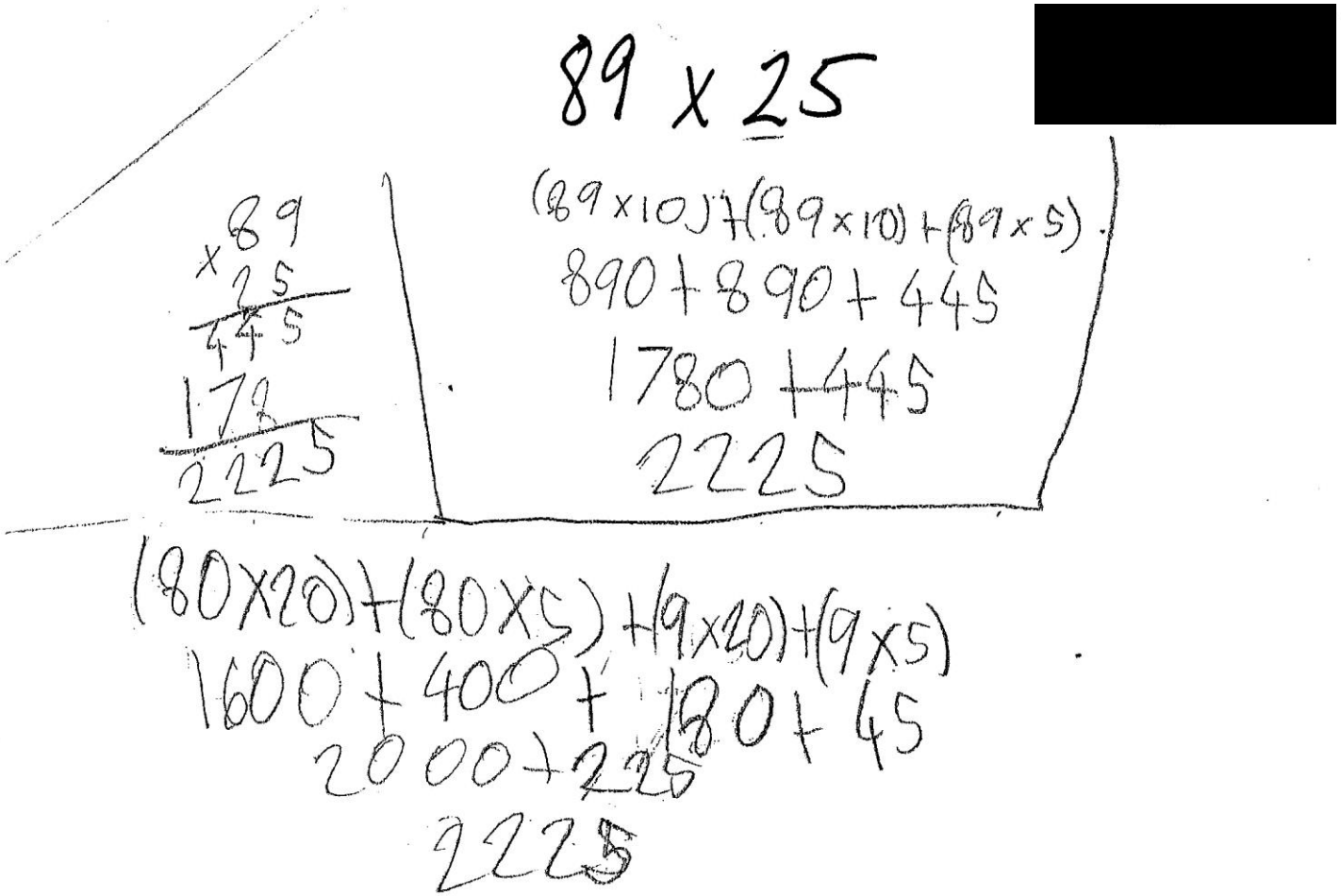
Student – Example D



Student – Example E



Student – Example F



Let's Compare Algorithms

$$\begin{array}{r} {\scriptstyle 1} \\ \not{4} \\ 89 \\ \times 25 \\ \hline 445 \\ +1780 \\ \hline 2225 \end{array}$$

89 x 25
=(80+9) (20+5)
=(80 x 20) + (80 x 5) + (9 x 20) + (9 x 5)
=1,600 + 400 + 180 + 45
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Traditional algorithms…

- Create misunderstandings
- Focuses on the digits, not the value of the numbers.

Distributive Property….

- Is Conceptual
- Promotes thinking
- Prepares children for Algebra
- Is a big idea that connects different aspects of multiplication.

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Let’s try another problem:

$$7 \times 4\frac{1}{9}$$

Is this how you solved it?

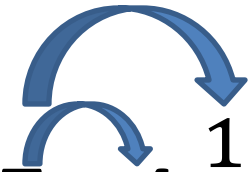
$$7 \times 4\frac{1}{9}$$

$$= \frac{7}{1} \times \frac{37}{9}$$

$$= \frac{259}{9}$$

$$= 28\frac{7}{9}$$

Did You Know You Can Solve it with Distribution?



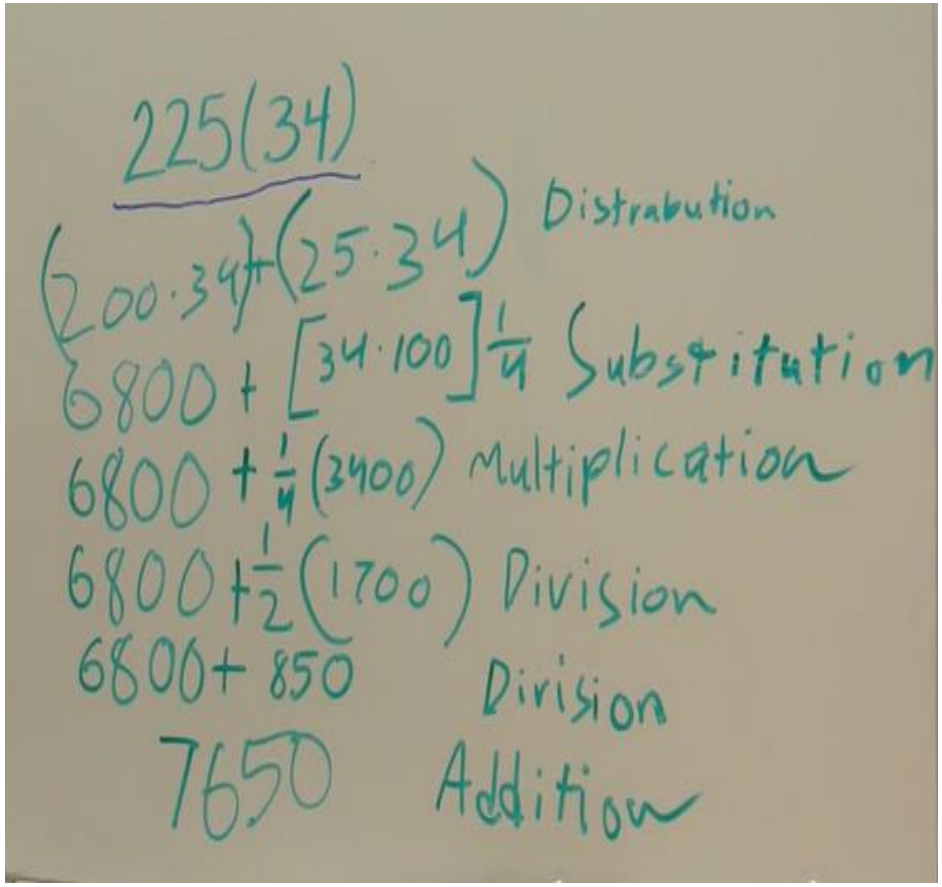
$$7 \times 4\frac{1}{9}$$

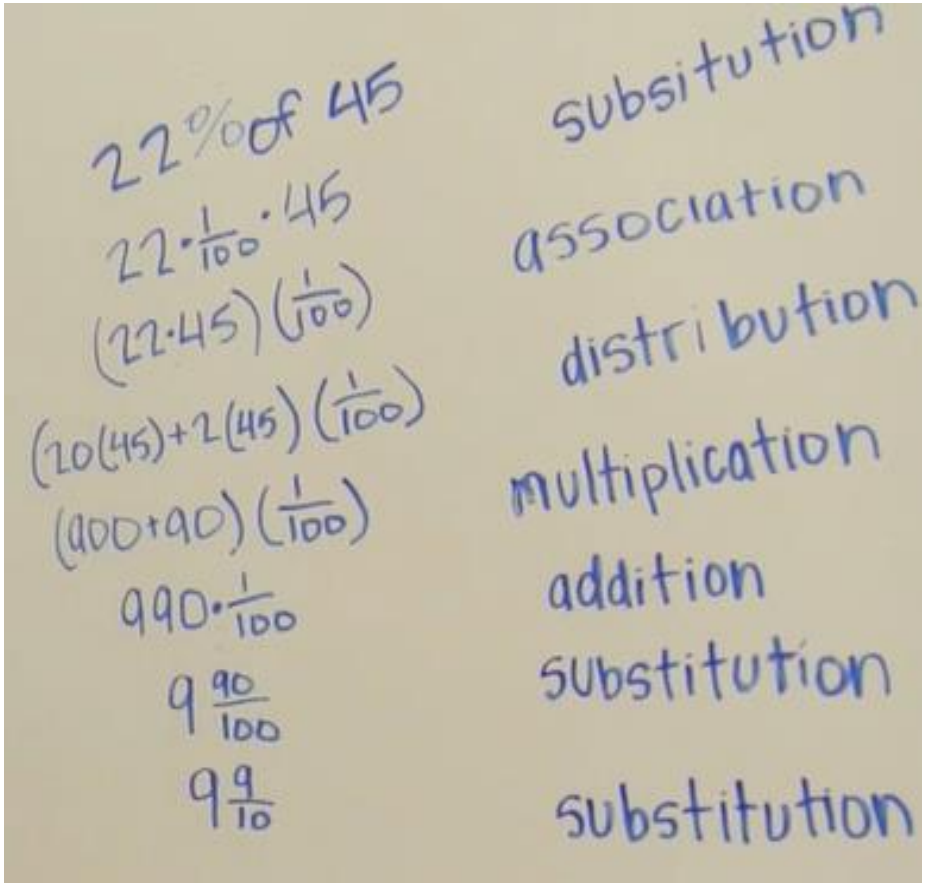
$$= 7(4) + 7\left(\frac{1}{9}\right)$$

$$= 28 + \frac{7}{9}$$

$$= 28\frac{7}{9}$$

Let's Compare





Learning mathematics involves learning ways of thinking. It involves learning powerful mathematical ideas rather than a collection of disconnected procedures for carrying out calculations. For many children, their weakness is learning procedures without understanding. These children will never achieve understanding by practicing procedures they don't understand. Our goal in teaching mathematics is to ensure all children learn with understanding. We have found that even young children are capable of learning powerful unifying ideas of mathematics that are the foundation of both arithmetic and algebra.

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