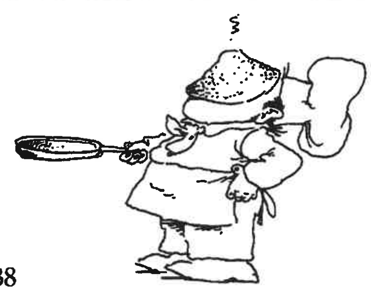
| | *Answers* |
|---|---|
| 23. The possible values of $(a,b)$ are (2002,1), (2001,2), (2000,3), . . . , (1003,1000), and (1002,1001). There are 1001 possible triangles.<br>A) 1001 B) 1002 C) 2002 D) 2003 | 23. A |
| 24. Rate's constant $\Rightarrow$ $\text{dist}_1:\text{hrs}_1 = \text{dist}_2:\text{hrs}_2 \Leftrightarrow \frac{k}{h} = \frac{h}{x}$. Cross-multiply to find $kx = h^2$. Solve for $x$ to see that $x = h^2/k$, choice D.<br>A) $\frac{k}{h}$ B) $\frac{h}{k}$ C) $\frac{k^2}{h}$ D) $\frac{h^2}{k}$  | 24. D |
| 25. $\frac{p}{100}(x) = 1 \Leftrightarrow \frac{x}{100} = \frac{1}{p}$, so $\frac{1}{p} \times \frac{1}{100} \times x = \frac{x^2}{10\,000}$.<br>A) $\frac{x}{10000}$ B) $\frac{x^2}{10000}$ C) $\frac{x}{100}$ D) $\frac{x^2}{100}$ | 25. B |
| 26. If $2003^{x^2+2x-35} = 1$, then $x^2+2x-35 = 0$, so $x = -7$ or 5.<br>A) -2 B) 2 C) -35 D) 35 | 26. C |
| 27. $5^3+91 = 6^3$, & $6^3+127 = 7^3$ so $a^3+125 = c^3$ has no pos. int. sols.<br>A) 0 B) 1 C) 2 D) 3 | 27. A |
| 28. Since $1/60 > 1/65 > 1/c$, 1/60 is the length of the hypotenuse. So, $1/c^2 + 1/65^2 = 1/60^2$ and $c = 156$.<br>A) $\sqrt{7825}$ B) $3900 \div \sqrt{7825}$<br>C) 70 D) 156 | 28. D |
| 29. If $a + \frac{1}{a} = 6$, then $\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2} = 6^2 = 36$. Since $a^2 + \frac{1}{a^2} = 34$, it must have taken me 34 attempts to learn how to flip flapjacks.<br>A) 34 B) 35 C) 36 D) 38  | 29. A |
| 30. $9^{4a} \times 49^{2b} = (3^2)^{4a} \times (7^2)^{2b} = (3^a)^8 \times (7^b)^4 = (2)^8 \times (5)^4$.<br>A) $2^4 \times 5^2$ B) $2^6 \times 5^6$ C) $2^8 \times 5^4$ D) $2^4 \times 5^8$ | 30. C |

*The end of the contest* A

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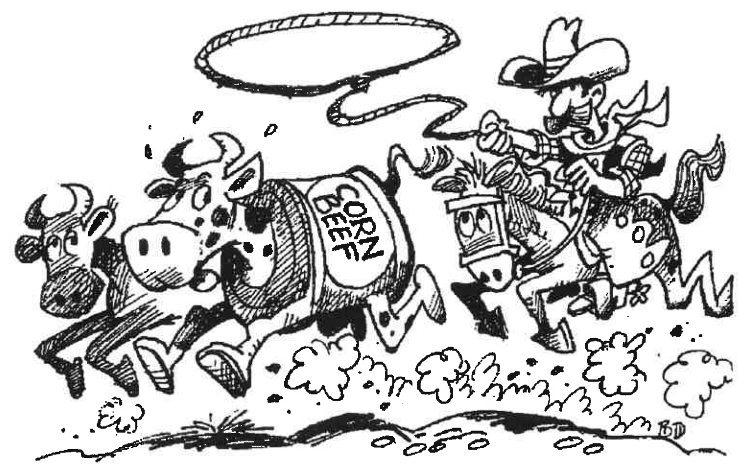
# Information & Solutions

*Spring, 2003*

**Contest Information**

**A**

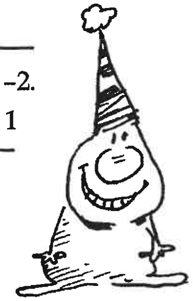
- **Solutions** Turn the page for detailed contest solutions (written in the question boxes) and letter answers (written in the *Answers* column to the right of each question).
- **Scores** Please remember that *this is a contest, not a test*—and there is no "passing" or "failing" score. Few students score as high as 30 points (75% correct). Students with half that, 15 points, *deserve commendation!*
- **Answers & Rating Scale** Turn to page 149 for the letter answers to each question and the rating scale for this contest.



1. If $x = 10$, then $2x^3 + 0x^2 + 0x + 3 = 2000 + 0 + 0 + 3 = 2003$.
A) 23 B) 203 C) 230 D) 2003
**1. D**

2. Since $x^2+3x-4 = (x+4)(x-1)$, the factors are $x+4$, $x-1$, and 1.
A) $x+4$ B) $x-1$ C) $x-4$ D) 1
**2. C**

3. Since $(n)(2003) = 2003+2003+2003+2003+2003 = 5\times 2003$, $n = 5$.
A) 5 B) 2001 C) 2002 D) 2003
**3. A**



4. $\frac{2}{x} = -1$ if $x = -2$, but $\frac{2}{x}$ is never integral if $x < -2$.
A) -2 B) 2 C) -1 D) 1
**4. A**

5. My unfolded hat's area is $s^2$.
Since its perimeter is $4s$, we get $\frac{s^2}{4s} = \frac{s}{4} = 4$.
Solving, we get $s = 16$.
A) 4 B) 8 C) 16 D) 64
**5. C**

6. $x+1-x+1+2+x-2+x = (x-x+x+x)+(1+1+2-2) = 2x + 2$.
A) $2x$ B) $2x+2$ C) 2 D) 6
**6. B**

7. The number of even integers between 1 and 2003 is the same as the number of odd integers between $4 = (1+3)$ and $(2003+3)$.
A) 2000 B) 2002 C) 2004 D) 2006
**7. D**

8. If $\frac{2001}{x} + \frac{2002}{x} + \frac{2003}{x} = 1$, then $\frac{6006}{x} = 1$ and $x = 6006$.
A) 6002 B) 6004 C) 6006 D) 6008
**8. C**

9. This is a right triangle with base $11 - 1 = 10$ and height $11 - 1 = 10$. Its area is $(10\times 10)/2 = 50$.
A) 50 B) 60.5 C) 100 D) 121
**9. A**


10. If each letter is a different positive odd integer, the least possible value of $\sqrt{s+t+o+p}$ is $\sqrt{1+3+5+7} = \sqrt{16} = 4$.
A) 2 B) 4 C) 8 D) 16
**10. B**

11. Since $y > 0$, we know that $\sqrt{y^8} = y^4$, $\sqrt{y^4} = y^2$, and $\sqrt{y^2} = y$. The correct answer is choice C.
A) $y^3$ B) $y^6$ C) $y^8$ D) $y^{16}$
**11. C**





12. The least possible sum is $1 + \frac{1}{1} = 2$.
A) 1 B) 1.5 C) 2 D) 2.5
**12. C**

13. $(x^2+y^2)^2 = (x^2+y^2)(x^2+y^2) = x^4+x^2y^2+y^2x^2+y^4 = x^4+2x^2y^2+y^4$.
A) $xy$ B) $2xy$ C) $x^2y^2$ D) $2x^2y^2$
**13. D**

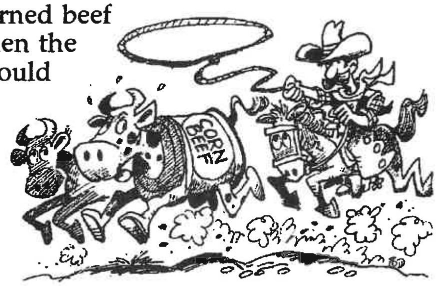
14. If $0 < x < 1$, then $x^{1000} < 1$. Since $4/\pi > 1$, $x^{1000} \neq 4/\pi$.
A) $\frac{1}{\pi}$ B) $\frac{2}{\pi}$ C) $\frac{3}{\pi}$ D) $\frac{4}{\pi}$
**14. D**

15. Since $(-5)^{2002} = 5^{2002}$, $5^{2003} \div (-5)^{2002} = 5^{2003} \div 5^{2002} = 5^1 = 5$.
A) 1 B) 5 C) -1 D) -5
**15. B**

16. The sum must be divisible by 3. Its square must be divisible by 9.
A) 2 B) 4 C) 9 D) 16
**16. C**

17. If $x = 0$, then $y = \pi(0)+\pi = \pi$, so $(0,-1)$ is not on the line.
A) $(-1,0)$ B) $(0,\pi)$ C) $(0,-1)$ D) $(1,2\pi)$
**17. C**

18. If I rounded up 1 can of corned beef for every 4 cans of soup, then the # of cans of corned beef would be 25% of the # of cans of soup. Of every 5 cans, 4 would be soup, so the # of cans of soup was 80% of the total number of cans.
A) 20 B) 25 C) 75 D) 80
**18. D**



19. $(x-1)^3(x+1)^2 = (x-1)[(x-1)(x+1)]^2 = (x-1)(x^2-1)^2$.
A) $(x-1)(x^2-1)^2$ B) $(x-1)(x^2+1)^2$
C) $(x+1)(x^2-1)^2$ D) $(x+1)(x^2+1)^2$
**19. A**

20. Subtract $x-y-13 = 0$ from $x+y-17 = 0$ to get $2y-4 = 0$, and $y = 2$.
A) 2 B) 13 C) 15 D) 17
**20. A**

21. If $(a,b) = (-3,-2)$, then $(x-a)(x+b) = (x+3)(x-2) = (x-2)(x+3)$.
A) $(-2,-3)$ B) $(2,-3)$ C) $(-3,2)$ D) $(-3,-2)$
**21. D**

22. $\dfrac{\frac{1}{x}+\frac{1}{y}}{x+y} = \left(\frac{1}{x}+\frac{1}{y}\right)\div(x+y) = \frac{x+y}{xy}\div(x+y) = \frac{x+y}{xy}\times\frac{1}{x+y} = \frac{1}{xy}$.
A) 1 B) $\frac{1}{xy}$ C) $xy$ D) $x + y$
**22. B**