Answer Keys:

Part I:

1. 269
2. 289
3. 2121
4. 16.7
5. 12
6. 1
7. 0.061
8. 15%
9. –6
10. $\sqrt{16+9}=5$
11. 133
12. 136
13. 73
14. 5040
15. ¢89
16. 2
17. – 4
18. 88.
19. 12/25
20. $90\frac{4}{25}$
21. 11
22. $\frac{-3}{5}$
23. 1331
24. 1
25. 74
26. 625
27. $-\sqrt{15}$
28. 20
29. 6
30. – 125
31. 8π - 6.
32. 72/5
33. –13
34. $x=3$.
35. $2\sqrt{2}$
36. 37/190.
37. 90
38. 1296
39. $-\frac{6}{7}$
40. 27_9

PART II

41. 1.

42. 14.548.

43. 2.

44. 7.

45. 1649.

46. 72.

47. 12.

48. 47.

49. 20.

50. \$9.

51. 1.29.

52. 1.

53. 45.

54. 20π.

55. $\frac{2016}{2017}$.

56. 4 %.

57. 1875.

58. 3.

59. $\frac{1}{2}$ hours.

60. August.

61. 9.

62. 1/20.

63. $2(a-b)$.

64. $\frac{15}{4}$.

65. 156.

66. 10.

67. 15.

68. 60.

69. 6.

70. 18°.

71. 2520.

72. 8/3.

73. 7.

74. 7600 seconds.

75. 182.5.

76. 5%.

77. 13.

78. 34 square units.

79. 11.

80. 9.

Solutions to Part II:

41. Solution: 1.

$(2^{2n}\cdot 5^2)^2 = 100{,}000 \Rightarrow \quad 2^{4n}\cdot 5^4 = 2^4\cdot 5^4 \Rightarrow \quad 2^{4n} = 2^4 \Rightarrow \quad 4n = 4 \Rightarrow \quad n = 1.$

42. Solution: 14.548.

$\sqrt{2} + \pi + e + 1.414 + 3.142 + 2.718 = 2(1.414 + 3.142 + 2.718) = 14.548.$

43. Solution: 2.

Square both sides of equation $\sqrt{5x} = x\sqrt{5}$: $5x = 5x^2 \quad \Rightarrow \quad x^2 - x = 0 \quad \Rightarrow$
$x(x-1) = 0$. $x = 1$ or $x = 0$. So we have two solutions.

44. Solution: 7.

$n \times 6! = 7! \quad \Rightarrow \quad n \times 6! = 7 \times 6! \Rightarrow \quad n = 7$

45. Solution: 1649.

$5 ☆ 4 = 5^4 + 4^5 = 625 + 1024 = 1649.$

46. Solution: 72.

Let the first bounce be x.

$(1/3)\,x = 24 \quad \Rightarrow \quad x = 72.$

47. Solution: 12.

As shown in the figure, we can write:

$2b + a = 16$ (1)

Applying Pythagorean Theorem to the right triangle ACD:

$b^2 - (\frac{a}{2})^2 = 4^2 \quad \Rightarrow \quad (2b)^2 - (a)^2 = 4\times 4^2$

$\Rightarrow \quad (2b + a)(2b - a) = 64$ (2)

Substituting (1) into (2): $2b - a = 4$ (3)

(1) − (2): $2a = 12$, which is exactly the area of the triangle.

48. Solution: 47.

By proportion: $\frac{50}{100} = \frac{x}{93} \quad \Rightarrow \quad x = \frac{50}{100}\times 93 = 46.5 \approx 47$.

49. Solution: 20.

We select 3 toppings from 6 toppings. Order does not matter.

$$\binom{6}{3} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

50. Solution: $9.

The total amount of money he paid is 11 + 29 = 40.
The total amount of money he received is 21 + 28 = 49.
The profit is 49 - 40 = $9.

51. Solution: 1.29.

$$\frac{15}{100} \times 0.86 = 1.29$$

52. Solution: 1.

Method 1:

$$\frac{x}{y} = \frac{3}{5} \qquad (1)$$

$$\frac{y}{z} = \frac{5}{3} \qquad (2)$$

(1) ÷ (2): $\frac{x}{z} = 1$.

Method 2:

$$\frac{x}{y} = \frac{3}{5} \qquad (1)$$

$$\frac{y}{z} = \frac{5}{3} \quad \Rightarrow \quad \frac{z}{y} = \frac{3}{5} \qquad (2)$$

Comparing (1) and (2) we see that $x = z$. So $\frac{x}{z} = 1$.

53. Solution: 45.

6! = 6 × 5 × 4 × 3 × 2 × 1. The greatest odd integer factor is 3 × 5 × 3 = 45.

54. Solution: 20π.

$$\frac{\pi}{4} d^2 = 100\pi^3 \quad \Rightarrow \quad d^2 = 400\pi^2 \quad \Rightarrow \quad d = 20\pi$$

55. Solution: $\frac{2016}{2017}$.

$$2015 \div 2015\frac{2015}{2016} = 2015 \div \frac{2015 \times 2016 + 2015}{2016} = 2015 \div \frac{2015 \times 2017}{2016}$$
$$= 2015 \times \frac{2016}{2015 \times 2017} = \frac{2016}{2017}$$

56. Solution: 4 %.
Let the legs be a and b.
$\frac{1.2a \times .08b}{2} = 0.96 \times \frac{ab}{2}$. The area of the triangle decreases by 1- 0.96 = 0.04 = 4%.

57. Solution: 1875.
Let x be the pieces of information can be entered.
We sue the proportion: $\frac{150}{12} = \frac{x}{120+30}$ $\Rightarrow$ $x = 1875$

58. Solution: 3.
Method 1:
$OP = OB - PB = OB - (AB - AP) = 6 - (8-5) = 3$.

Method 2:
We know that $AB = 8$ and $AP = 5$. So $PB = 3$. Since $OB = 6$, $OP = 3$.

59. Solution: $\frac{1}{2}$ hours.
$$\frac{\frac{2}{3}}{\frac{1}{3}} = \frac{1}{x} \qquad \Rightarrow \qquad x = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}.$$

60. Solution: August.
100 = 12 × 8 + 4.
Four months from now will be August.

61. Solution: 9.
$$\frac{9n}{n+1} > 8 \qquad \Rightarrow \qquad 9n > 8(n+1) \quad \Rightarrow \qquad n > 8.$$
The least possible value for n is 9.

62. Solution: 1/20.
There is only one card satisfying the requirement: 5. So the probability is 1/20.

63. Solution: $2(a-b)$.
$|b-a|+|a-c|+|c-b| = |a-b|+|a-c|+|c-b| = a-b+a-c+c-b = 2(a-b)$

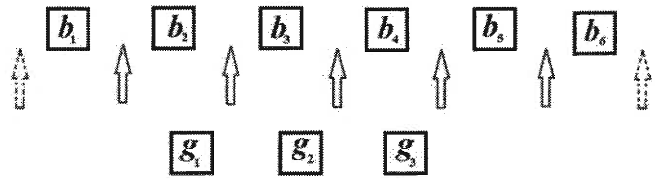
64. Solution: $\frac{15}{4}$.
Let v be the average speed for the entire round trip.
$v = \dfrac{2\times 10}{\frac{10}{3}+\frac{10}{5}} = \dfrac{15}{4}$.

65. Solution: 156.
Let a and b be the length and width of the rectangle, respectively.
$2(a+b)=50 \qquad \Rightarrow \qquad a+b=25$.
The greatest area is obtained when a and b are as close as possible. So $a = 12$ and $b = 13$.
The greatest area is $12 \times 13 = 156$.

66. Solution: 10.
$\frac{1}{2}\times\frac{1}{3}\times\frac{1}{5}\times 300 = 10$.

67. Solution: 15.
$\dfrac{\frac{2}{3}+\frac{1}{5}}{13} = \dfrac{\frac{13}{15}}{13} = \dfrac{1}{15}$. So $x = 15$,

68. Solution: 60.
We sit the boys first. Once the boys are seated, there are five spaces for three girls to choose $\binom{5}{3}$ (the two spaces at the ends do not count because each girl needs to sit between two boys) and there are 3! ways for them to rearrange themselves.



By the product rule, the answer is $\binom{5}{3} \times 3! = 60$.

69. Solution: 6.
$102 = 2 \times 51 = 2 \times 3 \times 17$
$114 = 2 \times 57 = 2 \times 3 \times 19$
The answer is $2 \times 3 = 6$.

70. Solution: 18°.
Method 1:
By proportion:
$$\frac{1}{1+3+6} \times 180 = 18°.$$

Method 2:
By solving the equation: $x + 3x + 6x = 180° \Rightarrow \quad x = 18°$.

71. Solution: 2520.
$\text{lcm} = 2^3 \times 3^2 \times 5 \times 7 = 2520$.

72. Solution: 8/3.
$$\sqrt{7\frac{1}{9}} = \sqrt{\frac{64}{9}} = \frac{8}{3}.$$

73. Solution: 7.
$119 = 13 \times 9 + 2$.
$q - r = 9 - 2 = 7$.

74. Solution: 7600 seconds.
1 hour = 3600 seconds.
$$2\frac{1}{9} \times 3600 = \frac{19}{9} \times 3600 = 19 \times 400 = 7600 \text{ seconds}.$$

75. Solution: 182.5.

$$\frac{4\times16\times365}{128}=182.5.$$

76. Solution: 5%.
Let the side be a.
$1.5a\times0.7a = 1.05a^2$. So the area of the original square is increased by 5 percent.

77. Solution: 13.
Let the sides of the rectangle be a and b.
$2(a+b)=34 \Rightarrow a+b=17 \Rightarrow (a+b)=17^2$ (1)
$ab=60 \Rightarrow 2ab=120$ (2)
(1) − (2): $a^2+b^2=169=13^2=c^2$.
The diagonal is 13 m.

78. Solution: 34 square units.
We view this solid from three directions: front view, side view, and top view. We get $2\times5+2\times5+2\times6=32$.

79. Solution: 11.
$x^2-y^2=1991 \Rightarrow x^2-y^2=(x-y)(x+y)=181\times11$.
Since 181 is a prime number and $x+y>x-y$, the greatest possible value of $x-y$ is 11.

80. Solution: 9.
The number of ways Alex draws x cards from box A is $\binom{18}{x}$

The number of ways Bob draws $3x-6$ cards from box A is $\binom{18}{3x-6}$.

So we set up the equation: $\binom{18}{x}=\binom{18}{3x-6}$ (1)

We have $x=3x-6$ (2)
or $x+3x-6=18$ (3)
with $0\le x\le18$ and $0\le 3x-6\le18$
Solving (2) and (3) we get: $x=3$ and $x=6$. Both are solutions of (1).
So the answer is $3+6=9$.